

Preface

As statisticians, we are constantly trying to make inferences about the underlying population from which data are observed. This includes estimation and prediction about the underlying population parameters from both complete and incomplete data. Recently, methodology for estimation and prediction from incomplete data has been found useful for what is known as “record-breaking data,” that is, data generated from setting new records. There has long been a keen interest in observing all kinds of records—in particular, sports records, financial records, flood records, and daily temperature records, to mention a few. The well-known *Guinness Book of World Records* is full of this kind of record information. As usual, beyond the general interest in knowing the last or current record value, the statistical problem of prediction of the next record based on past records has also been an important area of record research. Probabilistic and statistical models to describe behavior and make predictions from record-breaking data have been developed only within the last fifty or so years, with a relatively large amount of literature appearing on the subject in the last couple of decades. This book, written from a statistician’s perspective, is not a compilation of “records,” rather, it deals with the statistical issues of inference from a type of incomplete data, *record-breaking data*, observed as successive record values (maxima or minima) arising from a phenomenon or situation under study.

Prediction is just one aspect of statistical inference based on observed record values. Estimation of unknown parameters of statistical or probabilistic models fitted to observed records, or testing hypotheses about one or more such parameters, is as important as prediction. In addition, nonparametric models for record data and the associated inference procedures have been developed over the past decade and a half. This monograph concerns all of these types of inference based on record-breaking data. A few other books have been written on various aspects of records, and with the exception of the recent book by Arnold, Balakrishnan, and Nagaraja (1998), they have focused mostly on stochastic behavior of records, characterization, and prediction. Arnold et al. gave an excellent comprehensive review of most of the results on records, including some material on inference from record-breaking data. However, their chapter on inference from such data is somewhat brief, discussing mainly the estimation of parameters and not the more general problem of parametric and nonparametric inference from record-breaking data.

The main purpose of the present monograph is therefore to fill the inference gap mentioned above. Although inferences for parametric models are presented briefly, along with a summary of some of the results on prediction of future records, we focus on cataloging the

results on nonparametric inference from record-breaking data. Included is some material on parametric and nonparametric Bayesian models as well as a discussion of trend models. As a result, the material presented in this monograph provides a good supplement on inference from record-breaking data for a graduate course on the general topic of records. It should also serve as a valuable reference work on this topic for statisticians, probabilists, mathematicians, and other researchers.

The inference material presented here is intended to be somewhat self-contained, with many of the proofs and derivations of the important results included. For the basic estimation, prediction, and hypothesis testing results, a knowledge of at least first-year graduate-level mathematical statistics is assumed. For a good understanding of the asymptotic results on nonparametric function estimation in Chapters 4 and 5, basic knowledge of graduate-level probability theory and standard stochastic convergence theory is assumed.

After a brief introduction and background on stochastic characterizations of record values, Chapter 3 covers in some detail parametric inference from record-breaking data observed from exponential distributions and Weibull distributions. In addition, a summary of some of the results in the literature on prediction of future record values is given there for completeness. Chapter 4 begins the ideas of nonparametric inference procedures based on maximum likelihood methods and smooth nonparametric function estimation is described in detail in Chapter 5. Bayesian methods of inference are covered in Chapter 6, followed in Chapter 7 by models taking trends into account. The material in each chapter, after the background in Chapter 2, is somewhat independent, except that Chapters 4 and 5 should be covered together since the limiting behavior of the smooth function estimators relies on the asymptotic results in Chapter 4.

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