Contents

Preface **vii**

1 Simulating Random Numbers from a Uniform Distribution **1**
1.1 Uniform Integers and an Approximate Uniform Density 5
1.2 Simple Linear Congruential Generators 11
1.2.1 Structure in the Generated Numbers 14
1.2.2 Tests of Simple Linear Congruential Generators 20
1.2.3 Shuffling the Output Stream 21
1.2.4 Generation of Substreams in Simple Linear Congruential Generators 23
1.3 Computer Implementation of Simple Linear Congruential Generators 27
1.3.1 Ensuring Exact Computations 28
1.3.2 Restriction that the Output Be in the Open Interval $(0, 1)$ 29
1.3.3 Efficiency Considerations 30
1.3.4 Vector Processors 30
1.4 Other Linear Congruential Generators 31
1.4.1 Multiple Recursive Generators 32
1.4.2 Matrix Congruential Generators 34
1.4.3 Add-with-Carry, Subtract-with-Borrow, and Multiply-with-Carry Generators 35
1.5 Nonlinear Congruential Generators 36
1.5.1 Inversive Congruential Generators 36
1.5.2 Other Nonlinear Congruential Generators 37
1.6 Feedback Shift Register Generators 38
1.6.1 Generalized Feedback Shift Registers and Variations . . . 40
1.6.2 Skipping Ahead in GFSR Generators 43
1.7 Other Sources of Uniform Random Numbers 43
1.7.1 Generators Based on Cellular Automata 44
1.7.2 Generators Based on Chaotic Systems 45
1.7.3 Other Recursive Generators 45

1.7.4 Tables of Random Numbers . . . 46
1.8 Combining Generators . . . 46
1.9 Properties of Combined Generators . . . 48
1.10 Independent Streams and Parallel Random Number Generation . 51
1.10.1 Skipping Ahead with Combination Generators . . . 52
1.10.2 Different Generators for Different Streams . . . 52
1.10.3 Quality of Parallel Random Number Streams . . . 53
1.11 Portability of Random Number Generators . . . 54
1.12 Summary . . . 55
Exercises . . . 56

2 Quality of Random Number Generators 61
2.1 Properties of Random Numbers . . . 62
2.2 Measures of Lack of Fit . . . 64
2.2.1 Measures Based on the Lattice Structure . . . 64
2.2.2 Differences in Frequencies and Probabilities . . . 67
2.2.3 Independence . . . 70
2.3 Empirical Assessments . . . 71
2.3.1 Statistical Goodness-of-Fit Tests . . . 71
2.3.2 Comparisons of Simulated Results with Statistical Models in Physics . . . 86
2.3.3 Anecdotal Evidence . . . 86
2.3.4 Tests of Random Number Generators Used in Parallel . . 87
2.4 Programming Issues . . . 87
2.5 Summary . . . 87
Exercises . . . 88

3 Quasirandom Numbers 93
3.1 Low Discrepancy . . . 93
3.2 Types of Sequences . . . 94
3.2.1 Halton Sequences . . . 94
3.2.2 Sobol' Sequences . . . 96
3.2.3 Comparisons . . . 97
3.2.4 Variations . . . 97
3.2.5 Computations . . . 98
3.3 Further Comments . . . 98
Exercises . . . 100

4 Transformations of Uniform Deviates: General Methods 101
4.1 Inverse CDF Method . . . 102
4.2 Decompositions of Distributions . . . 109
4.3 Transformations that Use More than One Uniform Deviate . . . 111
4.4 Multivariate Uniform Distributions with Nonuniform Marginals . 112
4.5 Acceptance/Rejection Methods . . . 113
4.6 Mixtures and Acceptance Methods . . . 125

4.7 Ratio-of-Uniforms Method . . . 129
4.8 Alias Method . . . 133
4.9 Use of the Characteristic Function . . . 136
4.10 Use of Stationary Distributions of Markov Chains . . . 137
4.11 Use of Conditional Distributions . . . 149
4.12 Weighted Resampling . . . 149
4.13 Methods for Distributions with Certain Special Properties . . . 150
4.14 General Methods for Multivariate Distributions . . . 155
4.15 Generating Samples from a Given Distribution . . . 159
Exercises . . . 159

5 Simulating Random Numbers from Specific Distributions 165
5.1 Modifications of Standard Distributions . . . 167
5.2 Some Specific Univariate Distributions . . . 170
5.2.1 Normal Distribution . . . 171
5.2.2 Exponential, Double Exponential, and Exponential Power Distributions . . . 176
5.2.3 Gamma Distribution . . . 178
5.2.4 Beta Distribution . . . 183
5.2.5 Chi-Squared, Student's t, and F Distributions . . . 184
5.2.6 Weibull Distribution . . . 186
5.2.7 Binomial Distribution . . . 187
5.2.8 Poisson Distribution . . . 188
5.2.9 Negative Binomial and Geometric Distributions . . . 188
5.2.10 Hypergeometric Distribution . . . 189
5.2.11 Logarithmic Distribution . . . 190
5.2.12 Other Specific Univariate Distributions . . . 191
5.2.13 General Families of Univariate Distributions . . . 193
5.3 Some Specific Multivariate Distributions . . . 197
5.3.1 Multivariate Normal Distribution . . . 197
5.3.2 Multinomial Distribution . . . 198
5.3.3 Correlation Matrices and Variance-Covariance Matrices . 198
5.3.4 Points on a Sphere . . . 201
5.3.5 Two-Way Tables . . . 202
5.3.6 Other Specific Multivariate Distributions . . . 203
5.3.7 Families of Multivariate Distributions . . . 208
5.4 Data-Based Random Number Generation . . . 210
5.5 Geometric Objects . . . 212
Exercises . . . 213

6 Generation of Random Samples, Permutations, and Stochastic Processes 217
6.1 Random Samples . . . 217
6.2 Permutations . . . 220
6.3 Limitations of Random Number Generators . . . 220

6.4 Generation of Nonindependent Samples . . . 221
6.4.1 Order Statistics . . . 221
6.4.2 Censored Data . . . 223
6.5 Generation of Nonindependent Sequences . . . 224
6.5.1 Markov Process . . . 224
6.5.2 Nonhomogeneous Poisson Process . . . 225
6.5.3 Other Time Series Models . . . 226
Exercises . . . 227

7 Monte Carlo Methods 229
7.1 Evaluating an Integral . . . 230
7.2 Sequential Monte Carlo Methods . . . 233
7.3 Experimental Error in Monte Carlo Methods . . . 235
7.4 Variance of Monte Carlo Estimators . . . 236
7.5 Variance Reduction . . . 239
7.5.1 Analytic Reduction . . . 240
7.5.2 Stratified Sampling and Importance Sampling . . . 241
7.5.3 Use of Covariates . . . 245
7.5.4 Constrained Sampling . . . 248
7.5.5 Stratification in Higher Dimensions: Latin Hypercube Sampling . . . 248
7.6 The Distribution of a Simulated Statistic . . . 249
7.7 Computational Statistics . . . 250
7.7.1 Monte Carlo Methods for Inference . . . 251
7.7.2 Bootstrap Methods . . . 252
7.7.3 Evaluating a Posterior Distribution . . . 255
7.8 Computer Experiments . . . 256
7.9 Computational Physics . . . 257
7.10 Computational Finance . . . 261
Exercises . . . 271

8 Software for Random Number Generation 283
8.1 The User Interface for Random Number Generators . . . 285
8.2 Controlling the Seeds in Monte Carlo Studies . . . 286
8.3 Random Number Generation in Programming Languages . . . 286
8.4 Random Number Generation in IMSL Libraries . . . 288
8.5 Random Number Generation in S-Plus and R . . . 291
Exercises . . . 295

9 Monte Carlo Studies in Statistics 297
9.1 Simulation as an Experiment . . . 298
9.2 Reporting Simulation Experiments . . . 300
9.3 An Example . . . 301
Exercises . . . 310

A Notation and Definitions 313

B Solutions and Hints for Selected Exercises **323**

Bibliography **331**
Literature in Computational Statistics 332
World Wide Web, News Groups, List Servers, and Bulletin Boards . . 334
References for Software Packages 336
References to the Literature 336

Author Index **371**

Subject Index **377**