

# Contents

<b>Foreword</b>	<b>vii</b>
<b>Preface</b>	<b>xi</b>
<b>1 Brief History of Test Theory and Design</b>	<b>1</b>
1.1 Classical Test Design . . . . .	2
1.1.1 Standardized Testing in Psychology . . . . .	2
1.1.2 Classical Test Theory . . . . .	4
1.1.3 Discussion . . . . .	8
1.2 Modern Test Design . . . . .	9
1.2.1 New Notion of Standardization . . . . .	9
1.2.2 Item-Response Theory . . . . .	11
1.2.3 Item Calibration and Ability Measurement . . . . .	14
1.2.4 Test and Item Information Functions . . . . .	16
1.2.5 Test Characteristic Function . . . . .	17
1.2.6 Comparison Between Classical and IRT Parameters . . . . .	19
1.2.7 Ability Scale and Item Mapping . . . . .	19
1.2.8 Birnbaum Approach to Test Design . . . . .	21
1.3 Test Design in This Book . . . . .	23
1.3.1 Four Modes of Test Assembly . . . . .	24
1.3.2 Choice of Test Assembly Modes . . . . .	26
1.4 An Application of Integer Programming to Test Assembly . . . . .	26

1.5	Literature . . . . .	28
1.6	Summary . . . . .	30
1.7	Exercises . . . . .	31
<b>2</b>	<b>Formulating Test Specifications</b>	<b>33</b>
2.1	Examples of Test Specifications . . . . .	34
2.2	Classification of Attributes . . . . .	36
2.2.1	Type of Attribute . . . . .	36
2.2.2	Level of Attribute . . . . .	37
2.3	Constraints and Objectives . . . . .	38
2.4	Standard Form of the Set of Test Specifications . . . . .	40
2.4.1	Number of Objective Functions . . . . .	40
2.4.2	Number of Constraints . . . . .	41
2.5	Literature . . . . .	42
2.6	Summary . . . . .	43
2.7	Exercises . . . . .	44
<b>3</b>	<b>Modeling Test-Assembly Problems</b>	<b>47</b>
3.1	Identifying Decision Variables . . . . .	48
3.2	Modeling Constraints . . . . .	51
3.2.1	Quantitative Constraints . . . . .	51
3.2.2	Categorical Constraints . . . . .	55
3.2.3	Logical Constraints . . . . .	59
3.2.4	Checking Constraints . . . . .	61
3.3	Formulating Objective Functions . . . . .	64
3.3.1	Quantitative Objective Functions . . . . .	64
3.3.2	Categorical Objective Functions . . . . .	66
3.3.3	Objective Functions with Goal Values . . . . .	67
3.3.4	Multiobjective Test Assembly . . . . .	68
3.3.5	Nonlinear Objectives . . . . .	72
3.4	Literature . . . . .	72
3.5	Summary . . . . .	72
3.6	Exercises . . . . .	74
<b>4</b>	<b>Solving Test-Assembly Problems</b>	<b>77</b>
4.1	Standard Model for a Single Test . . . . .	78
4.1.1	Checking Interactions Between the Objective Function and Constraints . . . . .	80
4.2	Branch-and-Bound Search . . . . .	81
4.2.1	Tree Search . . . . .	82
4.2.2	Implementation Decisions . . . . .	84
4.2.3	Problem Size and Solution Time . . . . .	85
4.2.4	A Useful Approximation . . . . .	86
4.2.5	Software . . . . .	87
4.3	Network-Flow Approximation . . . . .	87

4.4	Constructive Heuristics . . . . .	89
4.4.1	Greedy Heuristics . . . . .	89
4.4.2	Luecht Heuristic . . . . .	91
4.4.3	Swanson-Stocking Heuristic . . . . .	91
4.5	Local Search Heuristics . . . . .	92
4.5.1	Genetic Algorithms . . . . .	93
4.5.2	Simulated Annealing . . . . .	94
4.6	Simultaneous and Sequential Optimization . . . . .	96
4.7	Optimal Design Approach . . . . .	98
4.8	Literature . . . . .	101
4.9	Summary . . . . .	102
<b>5</b>	<b>Models for Assembling Single Tests</b>	<b>105</b>
5.1	IRT-Based Test Assembly . . . . .	106
5.1.1	Absolute and Relative Targets . . . . .	107
5.1.2	Methods for Specifying Targets for Information Functions . . . . .	108
5.1.3	Assembling Tests for Absolute Targets . . . . .	110
5.1.4	Assembling Tests for Relative Targets . . . . .	113
5.1.5	Cutoff Scores . . . . .	114
5.1.6	Empirical Examples . . . . .	114
5.2	Classical Test Assembly . . . . .	115
5.2.1	Maximizing Test Reliability . . . . .	117
5.2.2	Maximizing Predictive Validity . . . . .	118
5.2.3	Constraining Test Reliability . . . . .	119
5.2.4	Empirical Example . . . . .	120
5.3	Matching Observed-Score Distributions . . . . .	121
5.3.1	Conditions on the Response Functions . . . . .	122
5.3.2	Constraints in the Test-Assembly Model . . . . .	123
5.3.3	Discussion . . . . .	124
5.3.4	Empirical Examples . . . . .	124
5.4	Item Matching . . . . .	125
5.4.1	Matching Items in a Reference Test . . . . .	129
5.4.2	Test Splitting . . . . .	131
5.4.3	Discussion . . . . .	133
5.4.4	Empirical Example . . . . .	133
5.5	Literature . . . . .	135
5.6	Summary . . . . .	136
5.7	Exercises . . . . .	137
<b>6</b>	<b>Models for Assembling Multiple Tests</b>	<b>139</b>
6.1	Sequential Assembly . . . . .	141
6.1.1	Heuristic Correction . . . . .	142
6.2	Simultaneous Assembly . . . . .	142
6.2.1	Item Overlap . . . . .	144

6.2.2	Controlling Targets Through Constraints . . . . .	145
6.3	Big-Shadow-Test Method . . . . .	146
6.3.1	Discussion . . . . .	150
6.4	Alternative Backup Methods . . . . .	151
6.5	Optimizing BIB Designs . . . . .	152
6.6	Empirical Examples . . . . .	155
6.7	Literature . . . . .	159
6.8	Summary . . . . .	161
6.9	Exercises . . . . .	162
<b>7</b>	<b>Models for Assembling Tests with Item Sets</b>	<b>165</b>
7.1	Simultaneous Selection of Items and Stimuli . . . . .	166
7.2	Power-Set Method . . . . .	170
7.3	Edited-Set Method . . . . .	174
7.4	Pivot-Item Method . . . . .	174
7.5	Two-Stage Method . . . . .	175
7.5.1	Stage 1: Selection of Stimuli . . . . .	175
7.5.2	Stage 2: Selection of Items from Sets . . . . .	178
7.5.3	Alternative Version . . . . .	179
7.6	Empirical Example . . . . .	179
7.7	Literature . . . . .	185
7.8	Summary . . . . .	185
7.9	Exercises . . . . .	186
<b>8</b>	<b>Models for Assembling Tests</b>	
	<b>Measuring Multiple Abilities</b>	<b>189</b>
8.1	Different Cases of Multidimensional Testing . . . . .	190
8.1.1	Both Abilities Intentional . . . . .	190
8.1.2	One Nuisance Ability . . . . .	191
8.1.3	Composite Ability . . . . .	191
8.1.4	Simple Structure of Multidimensional Abilities . . . . .	192
8.1.5	Simple Structure of Unidimensional Abilities . . . . .	192
8.2	Variance Functions . . . . .	192
8.3	Linearization of the Problem . . . . .	194
8.3.1	Linear Decomposition . . . . .	194
8.3.2	Linear Approximation . . . . .	197
8.4	Main Models . . . . .	197
8.4.1	Model for Relative Targets . . . . .	198
8.4.2	Model for Absolute Targets . . . . .	200
8.4.3	Applications to Different Cases . . . . .	200
8.5	Alternative Objectives	
	for Multidimensional Test Assembly . . . . .	203
8.5.1	Matching Observed-Score Distributions . . . . .	203
8.5.2	Item Matching . . . . .	204
8.5.3	Other Generalizations of Unidimensional Problems . . . . .	204

8.6	Empirical Example . . . . .	204
8.7	Literature . . . . .	207
8.8	Summary . . . . .	207
8.9	Exercises . . . . .	209
<b>9</b>	<b>Models for Adaptive Test Assembly</b>	<b>211</b>
9.1	Shadow-Test Approach . . . . .	213
9.1.1	Random Test Length . . . . .	214
9.1.2	Fixed Test Length . . . . .	214
9.1.3	Definition of Shadow Tests . . . . .	216
9.1.4	Standard Model for a Shadow Test . . . . .	217
9.1.5	Calculating Shadow Tests . . . . .	218
9.1.6	Empirical Example . . . . .	219
9.1.7	Discussion . . . . .	221
9.2	Alternative Objective Functions . . . . .	222
9.2.1	Kullback-Leibler Information . . . . .	223
9.2.2	Bayesian Item-Selection Criteria . . . . .	223
9.3	Adaptive Testing with Item Sets . . . . .	224
9.4	Controlling Item Exposure . . . . .	225
9.4.1	Alpha Stratification . . . . .	225
9.4.2	Sympson-Hetter Method . . . . .	229
9.4.3	Multiple-Shadow-Test Approach . . . . .	230
9.4.4	Method with Ineligibility Constraints . . . . .	233
9.5	Controlling the Speededness of the Test . . . . .	235
9.5.1	Response-Time Model . . . . .	237
9.5.2	Ability and Speed as Intentional Factors . . . . .	238
9.5.3	Speed as a Nuisance Factor . . . . .	239
9.6	Reporting Scores on a Reference Test . . . . .	241
9.7	Multidimensional Adaptive Test Assembly . . . . .	248
9.7.1	Minimizing Error Variances . . . . .	248
9.7.2	Computational Aspects . . . . .	251
9.7.3	Maximizing Kullback-Leibler Information . . . . .	251
9.7.4	Empirical Examples . . . . .	252
9.8	Final Comments . . . . .	253
9.9	Literature . . . . .	257
9.10	Summary . . . . .	259
9.11	Exercises . . . . .	261
<b>10</b>	<b>Designing Item Pools for Programs with Fixed Tests</b>	<b>265</b>
10.1	Definition of Design Space . . . . .	266
10.2	Programs with Parallel Forms of a Single Test . . . . .	268
10.2.1	Standard Design Model . . . . .	268
10.3	Programs with Parallel Forms of Multiple Tests . . . . .	270
10.3.1	Simultaneous Model . . . . .	271
10.3.2	Item Overlap . . . . .	272

10.3.3	Model with Aggregated Bounds . . . . .	275
10.3.4	Discussion . . . . .	276
10.4	Cost Function . . . . .	276
10.4.1	Smoothing Cost Functions . . . . .	277
10.5	Item Sets . . . . .	278
10.5.1	Simultaneous Model . . . . .	278
10.5.2	Three-Stage Approach . . . . .	281
10.6	Calculating Solutions . . . . .	282
10.7	Dynamic Versions of Design Models . . . . .	284
10.7.1	Dynamic Models . . . . .	284
10.7.2	Item Author as Attribute . . . . .	286
10.7.3	Empirical Example . . . . .	287
10.8	Assembling an Operational Item Pool . . . . .	290
10.9	Final Comment . . . . .	291
10.10	Literature . . . . .	292
10.11	Summary . . . . .	293
10.12	Exercises . . . . .	294
<b>11</b>	<b>Designing Item Pools for Programs with Adaptive Tests</b>	<b>297</b>
11.1	Programs with a Single Adaptive Test . . . . .	298
11.1.1	Design Model for Shadow Tests . . . . .	298
11.1.2	Blueprint without Item-Exposure Control . . . . .	301
11.1.3	Blueprint with Marginal Item-Exposure Control . . . . .	302
11.1.4	Blueprint with Conditional Item-Exposure Control . . . . .	302
11.1.5	Empirical Example . . . . .	303
11.2	Programs with Multiple Adaptive Tests . . . . .	304
11.2.1	Different Tests from the Same Item Pool . . . . .	304
11.2.2	Same Test from Different Item Pools . . . . .	305
11.3	Item Sets . . . . .	305
11.3.1	Design Model . . . . .	305
11.3.2	Calculating the Blueprint . . . . .	308
11.4	Calculating Shadow Tests . . . . .	309
11.5	Some Remaining Topics . . . . .	309
11.5.1	Stratifying an Item Pool . . . . .	309
11.5.2	Empirical Example . . . . .	310
11.5.3	Assembling an Item Pool as a Set of Fixed Test Forms . . . . .	311
11.5.4	Empirical Example . . . . .	313
11.5.5	Assembling a System of Rotating Item Pools . . . . .	314
11.5.6	Empirical Example . . . . .	320
11.6	Literature . . . . .	323
11.7	Summary . . . . .	324
11.8	Exercises . . . . .	325
<b>12</b>	<b>Epilogue</b>	<b>327</b>

<b>Appendix 1: Basic Concepts in Linear Programming</b>	<b>333</b>
A1.1 Mathematical Programming . . . . .	333
A1.1.1 Linear Programming . . . . .	334
A1.1.2 Nonlinear Programming . . . . .	335
A1.1.3 Other Forms of Mathematical Programming . . . . .	335
A1.1.4 Constraints on Variables . . . . .	336
A1.2 Graphical Example . . . . .	337
A1.2.1 Problem . . . . .	337
A1.2.2 Graphical Representation . . . . .	338
A1.2.3 Number of Solutions . . . . .	340
A1.3 Simplex Method . . . . .	341
A1.4 Network-Flow Problems . . . . .	342
A1.5 Solving Integer Problems . . . . .	344
A1.6 Literature . . . . .	346
<b>Appendix 2: Example of a Test-Assembly Problem in <i>OPL Studio</i></b>	<b>347</b>
<b>Answers to Exercises</b>	<b>353</b>
<b>Bibliography</b>	<b>389</b>
<b>Index</b>	<b>403</b>