Contents

Preface v

Chapter 1. Differentiable Manifolds 1
1. Basic Definitions 1
2. Differentiable Maps 5
3. Tangent Vectors 6
4. The Derivative 8
5. The Inverse and Implicit Function Theorems 11
6. Submanifolds 12
7. Vector Fields 16
8. The Lie Bracket 19
9. Distributions and Frobenius Theorem 27
10. Multilinear Algebra and Tensors 29
11. Tensor Fields and Differential Forms 35
12. Integration on Chains 41
13. The Local Version of Stokes' Theorem 43
14. Orientation and the Global Version of Stokes' Theorem 45
15. Some Applications of Stokes' Theorem 51

Chapter 2. Fiber Bundles 57
1. Basic Definitions and Examples 57
2. Principal and Associated Bundles 60
3. The Tangent Bundle of S^n 65
4. Cross-Sections of Bundles 67
5. Pullback and Normal Bundles 69
6. Fibrations and the Homotopy Lifting/Covering Properties 73
7. Grassmannians and Universal Bundles 75

Chapter 3. Homotopy Groups and Bundles Over Spheres 81
1. Differentiable Approximations 81
2. Homotopy Groups 83
3. The Homotopy Sequence of a Fibration 88
4. Bundles Over Spheres 94
5. The Vector Bundles Over Low-Dimensional Spheres 97

Chapter 4. Connections and Curvature 103
1. Connections on Vector Bundles 103
2. Covariant Derivatives 109
3. The Curvature Tensor of a Connection 114

4. Connections on Manifolds 120
5. Connections on Principal Bundles 125

Chapter 5. Metric Structures 131
1. Euclidean Bundles and Riemannian Manifolds 131
2. Riemannian Connections 133
3. Curvature Quantifiers 141
4. Isometric Immersions 145
5. Riemannian Submersions 147
6. The Gauss Lemma 155
7. Length-Minimizing Properties of Geodesics 160
8. First and Second Variation of Arc-Length 166
9. Curvature and Topology 171
10. Actions of Compact Lie Groups 173

Chapter 6. Characteristic Classes 177
1. The Weil Homomorphism 178
2. Pontrjagin Classes 181
3. The Euler Class 184
4. The Whitney Sum Formula for Pontrjagin and Euler Classes 189
5. Some Examples 191
6. The Unit Sphere Bundle and the Euler Class 199
7. The Generalized Gauss-Bonnet Theorem 203
8. Complex and Symplectic Vector Spaces 207
9. Chern Classes 215

Bibliography 221

Index 223