

---

## Contents

Preface .....	v
---------------	---

---

### Part I: Compact Groups

---

1 Haar Measure .....	3
2 Schur Orthogonality .....	6
3 Compact Operators.....	17
4 The Peter-Weyl Theorem.....	21

---

### Part II: Lie Group Fundamentals

---

5 Lie Subgroups of $\mathrm{GL}(n, \mathbb{C})$ .....	29
6 Vector Fields .....	36
7 Left-Invariant Vector Fields .....	41
8 The Exponential Map .....	46
9 Tensors and Universal Properties .....	50
10 The Universal Enveloping Algebra .....	54
11 Extension of Scalars .....	58
12 Representations of $\mathfrak{sl}(2, \mathbb{C})$ .....	62
13 The Universal Cover .....	69

<b>14</b>	<b>The Local Frobenius Theorem</b>	79
<b>15</b>	<b>Tori</b>	86
<b>16</b>	<b>Geodesics and Maximal Tori</b>	94
<b>17</b>	<b>Topological Proof of Cartan's Theorem</b>	107
<b>18</b>	<b>The Weyl Integration Formula</b>	112
<b>19</b>	<b>The Root System</b>	117
<b>20</b>	<b>Examples of Root Systems</b>	127
<b>21</b>	<b>Abstract Weyl Groups</b>	136
<b>22</b>	<b>The Fundamental Group</b>	146
<b>23</b>	<b>Semisimple Compact Groups</b>	150
<b>24</b>	<b>Highest-Weight Vectors</b>	157
<b>25</b>	<b>The Weyl Character Formula</b>	162
<b>26</b>	<b>Spin</b>	175
<b>27</b>	<b>Complexification</b>	182
<b>28</b>	<b>Coxeter Groups</b>	189
<b>29</b>	<b>The Iwasawa Decomposition</b>	197
<b>30</b>	<b>The Bruhat Decomposition</b>	205
<b>31</b>	<b>Symmetric Spaces</b>	212
<b>32</b>	<b>Relative Root Systems</b>	236
<b>33</b>	<b>Embeddings of Lie Groups</b>	257

---

**Part III: Topics**

---

<b>34</b>	<b>Mackey Theory</b>	275
<b>35</b>	<b>Characters of <math>\mathrm{GL}(n, \mathbb{C})</math></b>	284
<b>36</b>	<b>Duality between <math>S_k</math> and <math>\mathrm{GL}(n, \mathbb{C})</math></b>	289

<b>37</b>	<b>The Jacobi-Trudi Identity</b>	297
<b>38</b>	<b>Schur Polynomials and <math>\mathrm{GL}(n, \mathbb{C})</math></b>	308
<b>39</b>	<b>Schur Polynomials and <math>S_k</math></b>	315
<b>40</b>	<b>Random Matrix Theory</b>	321
<b>41</b>	<b>Minors of Toeplitz Matrices</b>	331
<b>42</b>	<b>Branching Formulae and Tableaux</b>	339
<b>43</b>	<b>The Cauchy Identity</b>	347
<b>44</b>	<b>Unitary Branching Rules</b>	357
<b>45</b>	<b>The Involution Model for <math>S_k</math></b>	361
<b>46</b>	<b>Some Symmetric Algebras</b>	370
<b>47</b>	<b>Gelfand Pairs</b>	375
<b>48</b>	<b>Hecke Algebras</b>	384
<b>49</b>	<b>The Philosophy of Cusp Forms</b>	397
<b>50</b>	<b>Cohomology of Grassmannians</b>	428
	<b>References</b>	438
	<b>Index</b>	446