# Measurement Errors and Uncertainties

Third Edition

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## Theory and Practice

Third Edition

Translated by
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# Preface

The major objective of this book is to give methods for estimating errors and uncertainties of real measurements: measurements that are performed in industry, commerce, and experimental research.

This book is needed because the existing theory of measurement errors was historically developed as an abstract mathematical discipline. As a result, this theory allows estimation of uncertainties of some ideal measurements only and is not applicable to most practical cases. In particular, it is not applicable to single measurements. This situation did not bother mathematicians, whereas engineers, not being bold enough to assert that the mathematical theory of errors cannot satisfy their needs, solved their particular problems in one or another ad hoc manner.

Actually, any measurement of a physical quantity is not abstract, but it involves an entirely concrete procedure that is always implemented with concrete technical devices—measuring instruments—under concrete conditions. Therefore, to obtain realistic estimates of measurement uncertainties, mathematical methods must be supplemented with methods that make it possible to take into account data on properties of measuring instruments, the conditions under which measurements are performed, the measurement procedure, and other features of measurements.

The importance of the methods of estimating measurement inaccuracies for practice can scarcely be exaggerated. Indeed, in another stage of planning a measurement or using a measurement result, one must know its error limits or uncertainty. Inaccuracy of a measurement determines its quality and is related to its cost. Reliability of product quality control also depends on accuracy of measurements. Without estimating measurement inaccuracies, one cannot compare measurement results obtained by different authors. Finally, it is now universally recognized that the precision with which any calculation using experimental data is performed must be consistent with the accuracy of these data.

In this book, the entire hierarchy of questions pertaining to measurement errors and uncertainties is studied, a theory of measurement inaccuracy is developed, and specific recommendations are made for solving the basic problems arising in practice. In addition, methods are presented for calculating the errors of measuring instruments. The attention devoted to the properties of measuring instruments, taking into account their relations with measurement inaccuracies, is one highlight of this book.

This book is a product of my professional scientific experience accumulated over many years of work in instrumentation and metrology. From 1948 to 1964, I was involved in the investigation and development of various electric measuring instruments, including calibrating potentiometers and stabilizers, extremely sensitive dc voltage and current amplifiers, automatic plotters, and so on. This experience gave me a grip in understanding problems arising in real measurements. Then, in 1965, I organized, and until 1980 directed, a laboratory of theoretical metrology. I focused on the analysis and generalization of theoretical problems in metrology. In particular, because I discovered that a rift exists between theory and practice (as mentioned above), I concentrated on the problem of estimating measurement errors and uncertainties. The results achieved during these years formed the foundation of my book *Measurement Errors* [44]. Further work and new results led to the writing of this book.

This book was initially published under the title *Measurement Errors: Theory and Practice* and has since gone through several editions, each reflecting new results that I have obtained. The initial hardware edition recommended a way to calculate the inaccuracy of single measurements. The paperback edition that followed added new treatment of indirect measurements, notably, a way of accounting for dependencies between the components of the uncertainty of indirect measurements. The second edition offered a full analysis of the method of reduction for processing indirect measurement data. The analysis shows the great advantage of this method over the traditional one based on the Taylor's series. In particular, the method of reduction obviates the need for the calculation of correlation coefficients. This development is important because the calculation of the correlation coefficient is one of the most notorious stumbling blocks in estimating the inaccuracy of measurement results.

However, the method of reduction is applicable only to dependent indirect measurements such as the measurement of electrical resistance using a voltmeter and ammeter. For independent indirect measurements, such as the measurement of the density of a solid body, the traditional method with its shortcomings was still inevitable. Only recently did I find a better solution for processing independent indirect measurement data. I called it the method of transformation. This method supplements the method of reduction and thus completes the creation of the new theory of indirect measurements. In addition to removing the need to calculate the correlation coefficient, the new theory allows the construction of the confidence intervals and produces well-grounded estimates of the uncertainty of both types of indirect measurements. This new theory is presented in this third edition of the book.

This edition has 12 chapters. Chapter 1 contains general information on measurements and metrology. Although introductory, the chapter includes some questions that are solved or presented anew. Also partially introductory is Chapter 2, devoted to measuring instruments. However, a large portion of it presents analysis of methods of standardization of the metrological characteristics of measuring instruments, which are important for practice and necessary for estimating measurement errors and uncertainties. Statistical analysis of errors of several batches of various measuring instruments obtained by standards laboratories is given. The analysis shows that such data are statistically unstable and hence cannot be the basis for obtaining a distribution function of errors of measuring instruments. This important result has influenced the ways in which many problems are covered in this book.

The inaccuracy of measurements always has to be estimated based on indirect data by finding and then summing the elementary components of the inaccuracy. In Chapter 3, a general analysis of elementary errors of measurements is given. Also, the classification of elementary errors is presented and their mathematical models are introduced. Two important methods of constructing a convolution of distribution functions are presented. These methods are necessary for summing elementary errors.

Chapter 4 contains methods of mathematical statistics as applied to idealized multiple measurements. In essence, these methods constitute the classical theory of measurement errors. New to the third edition is the review of modern robust and nonparametric methods of measurement data processing.

In Chapter 5, real direct measurements are considered. It is shown that single measurements should be considered as the basic form of measurement. Various methods for estimating and combining systematic and random errors are considered, and a comparative analysis of these methods is given. Special attention is paid to taking into account the errors of measuring instruments. For instance, it is shown how the uncertainty of a measurement result decreases when more accurate information on the properties of measuring instruments is used. This chapter concludes with a step-by-step procedure for estimating errors and uncertainties of direct measurements.

Chapter 6 presents the new theory of indirect measurements including the method of transformation that is added in this edition. The current edition also expands the examples of indirect measurements to illustrate the new method. These examples are taken out from Chapter 6 and organized into a separate Chapter 7.

In Chapter 8, combined measurements are considered. The well-known least-squares algorithm is described in detail. The new theory of indirect measurements allowed us to eliminate here the category of simultaneous measurements.

Chapter 9 contains methods for combining measurement results. Such methods are necessary in the cases where the same measurand is measured in multiple stages or in different laboratories. Along with the traditional solution, which takes into consideration only random errors, Chapter 9 includes a method taking into account systematic errors as well.

In Chapters 10 and 11, I return to considering measuring instruments. Chapter 10 gives general methods for calculating their total errors that are useful during the development of the instruments. In Chapter 11, calibration methods that tie measuring instruments to corresponding standards are considered.

The current edition also adds Chapter 12 with concluding remarks. This chapter briefly reviews the history of measurement data processing and outlines some current open problems in the theory and practice of measurements. The chapter also discusses two recent documents produced by international standards bodies, which are of fundamental importance to metrology: The "International Vocabulary of Basic and General Terms in Metrology" [2] and the "Guide to the Expression of Uncertainty in Measurement" [1].

In addition to the new theory of indirect measurements, the third edition contains many clarifications and corrections to the text of the second edition. Also, the list of references is updated.

The book is targeted for practical use and, to this end, includes many concrete examples, many of which illustrate typical problems arising in the practice of measurements.

This book is intended for anyone who is concerned with measurements in any field of science or technology, who designs technological processes and chooses for them instruments having appropriate accuracy, and who designs and tests new measuring devices. I also believe this book will prove useful to many university and college students. Indeed, measurements are of such fundamental importance for modern science and engineering that every engineer and every scientist doing experimental research must know the basics of the theory of measurements and especially how to estimate their accuracy.

In conclusion, I would like to thank Dr. E. Richard Cohen for carefully reading the manuscript of the second edition of this book and for many useful comments.

I would like to also thank Dr. Abram Kagan, now Professor at the University of Maryland, College Park, for the many years of collaboration and friendship. This book benefited from our discussions on various mathematical problems in metrology.

The initial hardback edition of the book was translated by M. E. Alferieff. The additions and changes to the subsequent editions were translated or edited by my son, Dr. Michael Rabinovich. Beyond that, Michael provided support and assistance throughout my work on this book, from editing the book proposal to publishers to discussing new results and the presentation. This book would not be possible without his help.

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