

Contents

Preface	vii
1 Straightedge and compass	1
1.1 Euclid's construction axioms	2
1.2 Euclid's construction of the equilateral triangle	4
1.3 Some basic constructions	6
1.4 Multiplication and division	10
1.5 Similar triangles	13
1.6 Discussion	17
2 Euclid's approach to geometry	20
2.1 The parallel axiom	21
2.2 Congruence axioms	24
2.3 Area and equality	26
2.4 Area of parallelograms and triangles	29
2.5 The Pythagorean theorem	32
2.6 Proof of the Thales theorem	34
2.7 Angles in a circle	36
2.8 The Pythagorean theorem revisited	38
2.9 Discussion	42
3 Coordinates	46
3.1 The number line and the number plane	47
3.2 Lines and their equations	48
3.3 Distance	51
3.4 Intersections of lines and circles	53
3.5 Angle and slope	55
3.6 Isometries	57

3.7	The three reflections theorem	61
3.8	Discussion	63
4	Vectors and Euclidean spaces	65
4.1	Vectors	66
4.2	Direction and linear independence	69
4.3	Midpoints and centroids	71
4.4	The inner product	74
4.5	Inner product and cosine	77
4.6	The triangle inequality	80
4.7	Rotations, matrices, and complex numbers	83
4.8	Discussion	86
5	Perspective	88
5.1	Perspective drawing	89
5.2	Drawing with straightedge alone	92
5.3	Projective plane axioms and their models	94
5.4	Homogeneous coordinates	98
5.5	Projection	100
5.6	Linear fractional functions	104
5.7	The cross-ratio	108
5.8	What is special about the cross-ratio?	110
5.9	Discussion	113
6	Projective planes	117
6.1	Pappus and Desargues revisited	118
6.2	Coincidences	121
6.3	Variations on the Desargues theorem	125
6.4	Projective arithmetic	128
6.5	The field axioms	133
6.6	The associative laws	136
6.7	The distributive law	138
6.8	Discussion	140
7	Transformations	143
7.1	The group of isometries of the plane	144
7.2	Vector transformations	146
7.3	Transformations of the projective line	151
7.4	Spherical geometry	154

7.5	The rotation group of the sphere	157
7.6	Representing space rotations by quaternions	159
7.7	A finite group of space rotations	163
7.8	The groups \mathbb{S}^3 and \mathbb{RP}^3	167
7.9	Discussion	170
8	Non-Euclidean geometry	174
8.1	Extending the projective line to a plane	175
8.2	Complex conjugation	178
8.3	Reflections and Möbius transformations	182
8.4	Preserving non-Euclidean lines	184
8.5	Preserving angle	186
8.6	Non-Euclidean distance	191
8.7	Non-Euclidean translations and rotations	196
8.8	Three reflections or two involutions	199
8.9	Discussion	203
	References	213
	Index	215