Contents

1 Bases and Basic Sequences 1
1.1 Schauder bases 1
1.2 Examples: Fourier series 6
1.3 Equivalence of bases and basic sequences 10
1.4 Bases and basic sequences: discussion 15
1.5 Constructing basic sequences 19
1.6 The Eberlein-Šmulian Theorem 23
Problems 25

2 The Classical Sequence Spaces 29
2.1 The isomorphic structure of the ℓ_p-spaces and c_0 29
2.2 Complemented subspaces of ℓ_p $(1 \leq p < \infty)$ and c_0 33
2.3 The space ℓ_1 36
2.4 Convergence of series 38
2.5 Complementability of c_0 44
Problems 48

3 Special Types of Bases 51
3.1 Unconditional bases 51
3.2 Boundedly-complete and shrinking bases 53
3.3 Nonreflexive spaces with unconditional bases 59
3.4 The James space $\mathcal{J}$ 62
3.5 A litmus test for unconditional bases 66
Problems 69

4 Banach Spaces of Continuous Functions 73
4.1 Basic properties 73
4.2 A characterization of real $\mathcal{C}(K)$-spaces 75
4.3 Isometrically injective spaces 79
4.4 Spaces of continuous functions on uncountable compact metric spaces 87

4.5 Spaces of continuous functions on countable compact metric spaces . . . 95
Problems . . . 98

5 $L_1(\mu)$-Spaces and $\mathcal{C}(K)$-Spaces . . . 101
5.1 General remarks about $L_1(\mu)$-spaces . . . 101
5.2 Weakly compact subsets of $L_1(\mu)$ . . . 103
5.3 Weak compactness in $\mathcal{M}(K)$ . . . 112
5.4 The Dunford-Pettis property . . . 115
5.5 Weakly compact operators on $\mathcal{C}(K)$-spaces . . . 118
5.6 Subspaces of $L_1(\mu)$-spaces and $\mathcal{C}(K)$-spaces . . . 120
Problems . . . 122

6 The L_p-Spaces for $1 \leq p < \infty$ . . . 125
6.1 Conditional expectations and the Haar basis . . . 125
6.2 Averaging in Banach spaces . . . 131
6.3 Properties of L_1 . . . 142
6.4 Subspaces of L_p . . . 148
Problems . . . 161

7 Factorization Theory . . . 165
7.1 Maurey-Nikishin factorization theorems . . . 165
7.2 Subspaces of L_p for $1 \leq p < 2$ . . . 173
7.3 Factoring through Hilbert spaces . . . 180
7.4 The Kwapień-Maurey theorems for type-2 spaces . . . 187
Problems . . . 191

8 Absolutely Summing Operators . . . 195
8.1 Grothendieck's Inequality . . . 196
8.2 Absolutely summing operators . . . 205
8.3 Absolutely summing operators on $L_1(\mu)$-spaces . . . 213
Problems . . . 217

9 Perfectly Homogeneous Bases and Their Applications . . . 221
9.1 Perfectly homogeneous bases . . . 221
9.2 Symmetric bases . . . 227
9.3 Uniqueness of unconditional basis . . . 229
9.4 Complementation of block basic sequences . . . 231
9.5 The existence of conditional bases . . . 235
9.6 Greedy bases . . . 240
Problems . . . 244

10 ℓ_p-Subspaces of Banach Spaces 247
10.1 Ramsey theory 247
10.2 Rosenthal's ℓ_1 theorem 251
10.3 Tsirelson space 254
Problems 259

11 Finite Representability of ℓ_p-Spaces 263
11.1 Finite representability 263
11.2 The Principle of Local Reflexivity 272
11.3 Krivine's theorem 275
Problems 285

12 An Introduction to Local Theory 289
12.1 The John ellipsoid 289
12.2 The concentration of measure phenomenon 293
12.3 Dvoretzky's theorem 296
12.4 The complemented subspace problem 301
Problems 306

13 Important Examples of Banach Spaces 309
13.1 A generalization of the James space 309
13.2 Constructing Banach spaces via trees 314
13.3 Pełczyński's universal basis space 316
13.4 The James tree space 317

A Fundamental Notions 327

B Elementary Hilbert Space Theory 331

C Main Features of Finite-Dimensional Spaces 335

D Cornerstone Theorems of Functional Analysis 337
D.1 The Hahn-Banach Theorem 337
D.2 Baire's Theorem and its consequences 338

E Convex Sets and Extreme Points 341

F The Weak Topologies 343

G Weak Compactness of Sets and Operators 347

List of Symbols 349

References 353

Index 365