

Contents

Preface	vii
Chapter 1. Riemannian Metrics	1
1. Riemannian Manifolds and Maps	2
2. Groups and Riemannian Manifolds	5
3. Local Representations of Metrics	8
4. Doubly Warped Products	13
5. Exercises	17
Chapter 2. Curvature	21
1. Connections	22
2. The Connection in Local Coordinates	29
3. Curvature	32
4. The Fundamental Curvature Equations	41
5. The Equations of Riemannian Geometry	47
6. Some Tensor Concepts	51
7. Further Study	56
8. Exercises	56
Chapter 3. Examples	63
1. Computational Simplifications	63
2. Warped Products	64
3. Hyperbolic Space	74
4. Metrics on Lie Groups	77
5. Riemannian Submersions	82
6. Further Study	90
7. Exercises	90
Chapter 4. Hypersurfaces	95
1. The Gauss Map	95
2. Existence of Hypersurfaces	97
3. The Gauss-Bonnet Theorem	101
4. Further Study	107
5. Exercises	108
Chapter 5. Geodesics and Distance	111
1. Mixed Partial	112
2. Geodesics	116
3. The Metric Structure of a Riemannian Manifold	121
4. First Variation of Energy	126
5. The Exponential Map	130

6. Why Short Geodesics Are Segments	132
7. Local Geometry in Constant Curvature	134
8. Completeness	137
9. Characterization of Segments	139
10. Riemannian Isometries	143
11. Further Study	149
12. Exercises	149
Chapter 6. Sectional Curvature Comparison I	153
1. The Connection Along Curves	153
2. Second Variation of Energy	158
3. Nonpositive Sectional Curvature	162
4. Positive Curvature	169
5. Basic Comparison Estimates	173
6. More on Positive Curvature	176
7. Further Study	182
8. Exercises	183
Chapter 7. The Bochner Technique	187
1. Killing Fields	188
2. Hodge Theory	202
3. Harmonic Forms	205
4. Clifford Multiplication on Forms	213
5. The Curvature Tensor	221
6. Further Study	229
7. Exercises	229
Chapter 8. Symmetric Spaces and Holonomy	235
1. Symmetric Spaces	236
2. Examples of Symmetric Spaces	244
3. Holonomy	252
4. Curvature and Holonomy	256
5. Further Study	262
6. Exercises	263
Chapter 9. Ricci Curvature Comparison	265
1. Volume Comparison	265
2. Fundamental Groups and Ricci Curvature	273
3. Manifolds of Nonnegative Ricci Curvature	279
4. Further Study	290
5. Exercises	290
Chapter 10. Convergence	293
1. Gromov-Hausdorff Convergence	294
2. Hölder Spaces and Schauder Estimates	301
3. Norms and Convergence of Manifolds	307
4. Geometric Applications	318
5. Harmonic Norms and Ricci curvature	321
6. Further Study	330
7. Exercises	331

Chapter 11. Sectional Curvature Comparison II	333
1. Critical Point Theory	333
2. Distance Comparison	338
3. Sphere Theorems	346
4. The Soul Theorem	349
5. Finiteness of Betti Numbers	357
6. Homotopy Finiteness	365
7. Further Study	372
8. Exercises	372
Appendix. De Rham Cohomology	375
1. Lie Derivatives	375
2. Elementary Properties	379
3. Integration of Forms	380
4. Čech Cohomology	383
5. De Rham Cohomology	384
6. Poincaré Duality	387
7. Degree Theory	389
8. Further Study	391
Bibliography	393
Index	397