

# Contents

<b>Preface</b>	<b>vii</b>
<b>1 Background</b>	<b>1</b>
1.1 Euclidean Spaces . . . . .	1
1.2 Symmetric Matrices . . . . .	9
<b>2 Inequality Constraints</b>	<b>15</b>
2.1 Optimality Conditions . . . . .	15
2.2 Theorems of the Alternative . . . . .	23
2.3 Max-functions . . . . .	28
<b>3 Fenchel Duality</b>	<b>33</b>
3.1 Subgradients and Convex Functions . . . . .	33
3.2 The Value Function . . . . .	43
3.3 The Fenchel Conjugate . . . . .	49
<b>4 Convex Analysis</b>	<b>65</b>
4.1 Continuity of Convex Functions . . . . .	65
4.2 Fenchel Biconjugation . . . . .	76
4.3 Lagrangian Duality . . . . .	88
<b>5 Special Cases</b>	<b>97</b>
5.1 Polyhedral Convex Sets and Functions . . . . .	97
5.2 Functions of Eigenvalues . . . . .	104
5.3 Duality for Linear and Semidefinite Programming . . . . .	109
5.4 Convex Process Duality . . . . .	114
<b>6 Nonsmooth Optimization</b>	<b>123</b>
6.1 Generalized Derivatives . . . . .	123
6.2 Regularity and Strict Differentiability . . . . .	130
6.3 Tangent Cones . . . . .	137
6.4 The Limiting Subdifferential . . . . .	145

<b>7 Karush–Kuhn–Tucker Theory</b>	<b>153</b>
7.1 An Introduction to Metric Regularity . . . . .	153
7.2 The Karush–Kuhn–Tucker Theorem . . . . .	160
7.3 Metric Regularity and the Limiting Subdifferential . . . . .	166
7.4 Second Order Conditions . . . . .	172
<b>8 Fixed Points</b>	<b>179</b>
8.1 The Brouwer Fixed Point Theorem . . . . .	179
8.2 Selection and the Kakutani–Fan Fixed Point Theorem . . . . .	190
8.3 Variational Inequalities . . . . .	200
<b>9 More Nonsmooth Structure</b>	<b>213</b>
9.1 Rademacher’s Theorem . . . . .	213
9.2 Proximal Normals and Chebyshev Sets . . . . .	218
9.3 Amenable Sets and Prox-Regularity . . . . .	228
9.4 Partly Smooth Sets . . . . .	233
<b>10 Postscript: Infinite Versus Finite Dimensions</b>	<b>239</b>
10.1 Introduction . . . . .	239
10.2 Finite Dimensionality . . . . .	241
10.3 Counterexamples and Exercises . . . . .	244
10.4 Notes on Previous Chapters . . . . .	248
<b>11 List of Results and Notation</b>	<b>253</b>
11.1 Named Results . . . . .	253
11.2 Notation . . . . .	267
<b>Bibliography</b>	<b>275</b>
<b>Index</b>	<b>289</b>