Contents

Series Preface . . . . . **vii**
Preface . . . . . **ix**
List of Symbols . . . . . **xix**

1 Introduction to Structures **1**
1.1 Examples 1
1.1.1 A Simple Structure 1
1.1.2 A 2D Truss 2
1.1.3 A 3D Truss 2
1.1.4 A Beam 3
1.1.5 The Deep Space Network Antenna 3
1.1.6 The International Space Station Structure 6
1.2 Definition 6
1.3 Properties 7

2 Standard Models **13**
2.1 Models of a Linear System 14
2.1.1 State-Space Representation 14
2.1.2 Transfer Function 15
2.2 Second-Order Structural Models 16
2.2.1 Nodal Models 16
2.2.2 Modal Models 17
2.3 State-Space Structural Models 29
2.3.1 Nodal Models 29
2.3.2 Models in Modal Coordinates 31
2.3.3 Modal Models 35

3 Special Models 41
3.1 Models with Rigid-Body Modes 41
3.2 Models with Accelerometers 45
3.2.1 State-Space Representation 45
3.2.2 Second-Order Representation 48
3.2.3 Transfer Function 49
3.3 Models with Actuators 50
3.3.1 Model with Proof-Mass Actuators 50
3.3.2 Model with Inertial Actuators 53
3.4 Models with Small Nonproportional Damping 54
3.5 Generalized Model 58
3.5.1 State-Space Representation 59
3.5.2 Transfer Function 59
3.6 Discrete-Time Models 60
3.6.1 State-Space Representation 61
3.6.2 Transfer Function 63

4 Controllability and Observability 65
4.1 Definition and Properties 65
4.1.1 Continuous-Time Systems 66
4.1.2 Discrete-Time Systems 68
4.1.3 Relationship Between Continuous- and Discrete-Time Grammians 69
4.2 Balanced Representation 71
4.3 Balanced Structures with Rigid-Body Modes 73
4.4 Input and Output Gains 74
4.5 Controllability and Observability of a Structural Modal Model . . . 76
4.5.1 Diagonally Dominant Grammians 76
4.5.2 Closed-Form Grammians 79
4.5.3 Approximately Balanced Structure in Modal Coordinates . . . 80
4.6 Controllability and Observability of a Second-Order Modal Model . . 85
4.6.1 Grammians 85
4.6.2 Approximately Balanced Structure in Modal Coordinates . . . 87
4.7 Three Ways to Compute Hankel Singular Values 91
4.8 Controllability and Observability of the Discrete-Time Structural Model 91
4.9 Time-Limited Grammians 94
4.10 Frequency-Limited Grammians 99
4.11 Time- and Frequency-Limited Grammians 103
4.12 Discrete-Time Grammians in Limited-Time and -Frequency Range 107

5 Norms 109
5.1 Norms of the Continuous-Time Systems 109
5.1.1 The H_2 Norm 109
5.1.2 The H_∞ Norm 111
5.1.3 The Hankel Norm 112

5.2 Norms of the Discrete-Time Systems . . . 113
5.2.1 The H_2 Norm . . . 113
5.2.2 The H_∞ Norm . . . 114
5.2.3 The Hankel Norm . . . 114
5.3 Norms of a Single Mode . . . 115
5.3.1 The H_2 Norm . . . 115
5.3.2 The H_∞ Norm . . . 117
5.3.3 The Hankel Norm . . . 118
5.3.4 Norm Comparison . . . 119
5.4 Norms of a Structure . . . 120
5.4.1 The H_2 Norm . . . 121
5.4.2 The H_∞ Norm . . . 121
5.4.3 The Hankel Norm . . . 123
5.5 Norms of a Structure with a Filter . . . 124
5.5.1 The H_2 Norm . . . 124
5.5.2 The H_∞ Norm . . . 126
5.5.3 The Hankel Norm . . . 127
5.6 Norms of a Structure with Actuators and Sensors . . . 127
5.6.1 The H_2 Norm . . . 128
5.6.2 The H_∞ Norm . . . 130
5.6.3 The Hankel Norm . . . 132
5.7 Norms of a Generalized Structure . . . 135
5.8 Norms of the Discrete-Time Structures . . . 137
5.8.1 The H_2 Norm . . . 138
5.8.2 The H_∞ Norm . . . 139
5.8.3 The Hankel Norm . . . 140
5.8.4 Norm Comparison . . . 140

6 Model Reduction . . . 143
6.1 Reduction Through Truncation . . . 143
6.2 Reduction Errors . . . 145
6.2.1 H_2 Model Reduction . . . 145
6.2.2 H_∞ and Hankel Model Reduction . . . 146
6.3 Reduction in the Finite-Time and -Frequency Intervals . . . 147
6.3.1 Reduction in the Finite-Time Interval . . . 148
6.3.2 Reduction in the Finite-Frequency Interval . . . 150
6.3.3 Reduction in the Finite-Time and -Frequency Intervals . . . 151
6.4 Structures with Rigid-Body Modes . . . 155
6.5 Structures with Actuators and Sensors . . . 159
6.5.1 Actuators and Sensors in a Cascade Connection . . . 159
6.5.2 Structure with Accelerometers . . . 161
6.5.3 Structure with Proof-Mass Actuators . . . 162
6.5.4 Structure with Inertial Actuators . . . 165

7 Actuator and Sensor Placement 167
7.1 Problem Statement 168
7.2 Additive Property of Modal Norms 168
7.2.1 The H_2 Norm 169
7.2.2 The H_∞ and Hankel Norms 169
7.3 Placement Indices and Matrices 170
7.3.1 H_2 Placement Indices and Matrices 170
7.3.2 H_∞ and Hankel Placement Indices and Matrices 172
7.3.3 Actuator/Sensor Indices and Modal Indices 173
7.4 Placement for Large Structures 180
7.4.1 Actuator Placement Strategy 182
7.4.2 Sensor Placement Strategy 182
7.5 Placement for a Generalized Structure 187
7.5.1 Structural Testing and Control 187
7.5.2 Sensor and Actuator Properties 189
7.5.3 Placement Indices and Matrices 192
7.5.4 Placement of a Large Number of Sensors 193
7.6 Simultaneous Placement of Actuators and Sensors 197

8 Modal Actuators and Sensors 203
8.1 Modal Actuators and Sensors Through Modal Transformations . . 204
8.1.1 Modal Actuators 204
8.1.2 Modal Sensors 208
8.2 Modal Actuators and Sensors Through Grammian Adjustment . . 213

9 System Identification 219
9.1 Discrete-Time Systems 220
9.2 Markov Parameters 221
9.3 Identification Algorithm 221
9.4 Determining Markov Parameters 224
9.5 Examples 226
9.5.1 A Simple Structure 226
9.5.2 The 2D Truss 230
9.5.3 The Deep Space Network Antenna 232

10 Collocated Controllers 235
10.1 A Low-Authority Controller 236
10.2 Dissipative Controller 237
10.3 Properties of Collocated Controllers 239
10.4 Root-Locus of Collocated Controllers 241
10.5 Collocated Controller Design Examples 245
10.5.1 A Simple Structure 245
10.5.2 The 2D Truss 246

11 LQG Controllers 249
11.1 Definition and Gains 250
11.2 The Closed-Loop System 253

11.3 The Balanced LQG Controller . . . 254
11.4 The Low-Authority LQG Controller . . . 255
11.5 Approximate Solutions of CARE and FARE . . . 257
11.6 Root-Locus . . . 260
11.7 Almost LQG-Balanced Modal Representation . . . 262
11.8 Three Ways to Compute LQG Singular Values . . . 264
11.9 The Tracking LQG Controller . . . 264
11.10 Frequency Weighting . . . 266
11.11 The Reduced-Order LQG Controller . . . 269
11.11.1 The Reduction Index . . . 269
11.11.2 The Reduction Technique . . . 271
11.11.3 Stability of the Reduced-Order Controller . . . 272
11.11.4 Performance of the Reduced-Order Controller . . . 274
11.11.5 Weights of Special Interest . . . 275
11.12 Controller Design Procedure . . . 276
11.13 Controller Design Examples . . . 277
11.13.1 A Simple Structure . . . 277
11.13.2 The 3D Truss . . . 279
11.13.3 The 3D Truss with Input Filter . . . 281
11.13.4 The Deep Space Network Antenna . . . 283

12 H_∞ and H_2 Controllers . . . 287
12.1 Definition and Gains . . . 288
12.2 The Closed-Loop System . . . 291
12.3 The Balanced H_∞ Controller . . . 292
12.4 The H_2 Controller . . . 294
12.4.1 Gains . . . 294
12.4.2 The Balanced H_2 Controller . . . 296
12.5 The Low-Authority H_∞ Controller . . . 296
12.6 Approximate Solutions of HCARE and HFARE . . . 298
12.7 Almost H_∞-Balanced Modal Representation . . . 300
12.8 Three Ways to Compute H_∞ Singular Values . . . 301
12.9 The Tracking H_∞ Controller . . . 301
12.10 Frequency Weighting . . . 301
12.11 The Reduced-Order H_∞ Controller . . . 304
12.11.1 The Reduction Index . . . 304
12.11.2 Closed-Loop Poles . . . 304
12.11.3 Controller Performance . . . 306
12.12 Controller Design Procedure . . . 307
12.13 Controller Design Examples . . . 308
12.13.1 A Simple Structure . . . 308
12.13.2 The 2D Truss . . . 310
12.13.3 Filter Implementation Example . . . 312
12.13.4 The Deep Space Network Antenna with Wind Disturbance Rejection Properties . . . 313

Appendices **317**

A Matlab Functions **319**

A.1 Transformation from an Arbitrary State-Space Representation to the Modal 1 State-Space Representation 320

A.2 Transformation from an Arbitrary State-Space Representation to the Modal 2 State-Space Representation 322

A.3 Transformation from Modal Parameters to the Modal 1 State-Space Representation 324

A.4 Transformation from Modal Parameters to the Modal 2 State-Space Representation 325

A.5 Transformation from Nodal Parameters to the Modal 1 State-Space Representation 326

A.6 Transformation from Nodal Parameters to the Modal 2 State-Space Representation 328

A.7 Determination of the Modal 1 State-Space Representation and the Time- and Frequency-Limited Grammians 329

A.8 Open-Loop Balanced Representation 331

A.9 H_2 Norm of a Mode 332

A.10 H_∞ Norm of a Mode 333

A.11 Hankel Norm of a Mode 333

A.12 LQG-Balanced Representation 334

A.13 H_∞-Balanced Representation 335

B Matlab Examples **337**

B.1 Example 2.5 337

B.2 Example 3.3 341

B.3 Example 4.11 342

B.4 Example 5.3 344

B.5 Example 6.7 347

B.6 Example 7.2 348

B.7 Example 8.1 353

B.8 Example 9.1 356

B.9 Example 10.4.2 359

B.10 Example 11.13.1 361

B.11 Example 12.13.2 365

C Structural Parameters **371**

C.1 Mass and Stiffness Matrices of the 2D Truss 371

C.2 Mass and Stiffness Matrices of the Clamped Beam Divided into 15 Finite Elements 373

C.3 State-Space Representation of the Deep Space Network Antenna . 376

References **379**

Index **389**