
Contents

Preface	V
1 Positioning on Earth and in Space	1
1.1 Introduction	1
1.2 Global Positioning System	2
1.2.1 Some Facts about GPS	2
1.2.2 The Theory Behind GPS	3
1.2.3 Dealing with Practical Difficulties	6
1.3 How Hydro-Québec Manages Lightning Strikes	12
1.3.1 Locating Lightning Strikes	12
1.3.2 Threshold and Quality of Detection	15
1.3.3 Long-Term Risk Management	18
1.4 Linear Shift Registers	19
1.4.1 The Structure of the Field \mathbb{F}_2^r	22
1.4.2 Proof of Theorem 1.4	24
1.5 Cartography	27
1.6 Exercises	36
References	43
2 Friezes and Mosaics	45
2.1 Friezes and Symmetries	48
2.2 Symmetry Group and Affine Transformations	52
2.3 The Classification Theorem	58
2.4 Mosaics	64
2.5 Exercises	67
References	83

3 Robotic Motion	85
3.1 Introduction	85
3.1.1 Moving a Solid in the Plane	87
3.1.2 Some Thoughts on the Number of Degrees of Freedom	89
3.2 Movements That Preserve Distances and Angles	91
3.3 Properties of Orthogonal Matrices	94
3.4 Change of Basis	103
3.5 Different Frames of Reference for a Robot	106
3.6 Exercises	111
References	117
4 Skeletons and Gamma-Ray Radiosurgery	119
4.1 Introduction	119
4.2 Definition of Two-Dimensional Region Skeletons	120
4.3 Three-Dimensional Regions	130
4.4 The Optimal Surgery Algorithm	132
4.5 A Numerical Algorithm	134
4.5.1 The First Part of the Algorithm	135
4.5.2 Second Part of the Algorithm	139
4.5.3 Proof of Proposition 4.17	140
4.6 Other Applications of Skeletons	142
4.7 The Fundamental Property of the Skeleton	143
4.8 Exercises	147
References	153
5 Savings and Loans	155
5.1 Banking Vocabulary	155
5.2 Compound Interest	156
5.3 A Savings Plan	159
5.4 Borrowing Money	161
5.5 Appendix: Mortgage Payment Tables	164
5.6 Exercises	168
References	171
6 Error-Correcting Codes	173
6.1 Introduction: Digitizing, Detecting and Correcting	173
6.2 The Finite Field \mathbb{F}_2	178
6.3 The $C(7, 4)$ Hamming Code	179
6.4 $C(2^k - 1, 2^k - k - 1)$ Hamming Codes	182
6.5 Finite Fields	185
6.6 Reed–Solomon Codes	193

6.7	Appendix: The Scalar Product and Finite Fields	198
6.8	Exercises	200
References		207
7	Public Key Cryptography	209
7.1	Introduction	209
7.2	A Few Tools from Number Theory	210
7.3	The Idea behind RSA	213
7.4	Constructing Large Primes	221
7.5	The Shor Factorization Algorithm	231
7.6	Exercises	234
References		239
8	Random-Number Generators	241
8.1	Introduction	241
8.2	Linear Shift Registers	245
8.3	\mathbb{F}_p -Linear Generators	248
8.3.1	The Case $p = 2$	248
8.3.2	A Lesson on Gambling Machines	253
8.3.3	The General Case	253
8.4	Combined Multiple Recursive Generators	255
8.5	Conclusion	257
8.6	Exercises	258
References		263
9	Google and the <i>PageRank</i> Algorithm	265
9.1	Search Engines	265
9.2	The Web and Markov Chains	268
9.3	An Improved <i>PageRank</i>	278
9.4	The Frobenius Theorem	281
9.5	Exercises	284
References		289
10	Why 44,100 Samples per Second?	291
10.1	Introduction	291
10.2	The Musical Scale	292
10.3	The Last Note (Introduction to Fourier Analysis)	296
10.4	The Nyquist Frequency and the Reason for 44,100	307
10.5	Exercises	317
References		323

11 Image Compression: Iterated Function Systems	325
11.1 Introduction	325
11.2 Affine Transformations in the Plane	327
11.3 Iterated Function Systems	330
11.4 Iterated Contractions and Fixed Points	336
11.5 The Hausdorff Distance	340
11.6 Fractal Dimension	345
11.7 Photographs as Attractors	350
11.8 Exercises	361
References	367
12 Image Compression: The JPEG Standard	369
12.1 Introduction	369
12.2 Zooming in on a JPEG-Compressed Digital Image	372
12.3 The Case of 2×2 Blocks	373
12.4 The Case of $N \times N$ Blocks	378
12.5 The JPEG Standard	388
12.6 Exercises	396
References	401
13 The DNA Computer	403
13.1 Introduction	403
13.2 Adleman's Hamiltonian Path Problem	405
13.3 Turing Machines and Recursive Functions	409
13.3.1 Turing Machines	409
13.3.2 Primitive Recursive Functions and Recursive Functions	416
13.4 Turing Machines and Insertion–Deletion Systems	426
13.5 NP-Complete Problems	430
13.5.1 The Hamiltonian Path Problem	430
13.5.2 Satisfiability	431
13.6 More on DNA Computers	435
13.6.1 The Hamiltonian Path Problem and Insertion–Deletion Systems	435
13.6.2 Current Limits	435
13.6.3 A Few Biological Explanations Concerning Adleman's Experiment	437
13.7 Exercises	441
References	445

14 Calculus of Variations	447
14.1 The Fundamental Problem of Calculus of Variations	448
14.2 Euler–Lagrange Equation	451
14.3 Fermat’s Principle	455
14.4 The Best Half-Pipe	457
14.5 The Fastest Tunnel	460
14.6 The Tautochrone Property of the Cycloid	465
14.7 An Isochronous Device	468
14.8 Soap Bubbles	471
14.9 Hamilton’s Principle	475
14.10 Isoperimetric Problems	479
14.11 Liquid Mirrors	486
14.12 Exercises	490
References	499
15 Science Flashes	501
15.1 The Laws of Reflection and Refraction	501
15.2 A Few Applications of Conics	508
15.2.1 A Remarkable Property of the Parabola	508
15.2.2 The Ellipse	518
15.2.3 The Hyperbola	520
15.2.4 A Few Clever Tools for Drawing Conics	521
15.3 Quadratic Surfaces in Architecture	521
15.4 Optimal Cellular Antenna Placement	528
15.5 Voronoi Diagrams	532
15.6 Computer Vision	537
15.7 A Brief Look at Computer Architecture	539
15.8 Regular Pentagonal Tiling of the Sphere	544
15.9 Laying Out a Highway	551
15.10 Exercises	552
References	567
Index	569