Contents

1. Scalar Derivatives in Euclidean Spaces 1
   - 1.1 Scalar Derivatives of Mappings in Euclidean Spaces 1
     - 1.1.1 Some Basic Results Concerning Skew-Adjoint Operators 2
     - 1.1.2 The Scalar Derivative and its Fundamental Properties 3
     - 1.1.3 Case $n = 2$. The Relation of the Scalar Derivative with the Complex Derivative 7
     - 1.1.4 Miscellanea Concerning Scalar Differentiability 9
     - 1.1.5 Characterization of Monotonicity by Scalar Derivatives 12
   - 1.2 Computational Formulae for the Scalar Derivative 15
     - 1.2.1 Scalar Derivatives and Directional Derivatives 15
     - 1.2.2 Applications 20
   - 1.3 Monotonicity, Scalar Differentiability, and Conformity 24
     - 1.3.1 The Coefficient of Conformity and the Conformal Derivative 25
     - 1.3.2 Monotone Vector Fields and Expansive Maps 27
2. Asymptotic Derivatives and Asymptotic Scalar Derivatives 31
   - 2.1 Asymptotic Differentiability in Banach Spaces 31
   - 2.2 Hyers–Ulam Stability and Asymptotic Derivatives 34
   - 2.3 Asymptotic Differentiability Along a Convex Cone in a Banach Space 45
   - 2.4 Asymptotic Differentiability in Locally Convex Spaces 49
   - 2.5 The Asymptotic Scalar Differentiability 64
   - 2.6 Some Applications 71

3. Scalar Derivatives in Hilbert Spaces 79
3.1 Calculus 79
3.1.1 Introduction 79
3.1.2 Some Basic Results Concerning Skew-Adjoint Operators 80
3.1.3 Scalar Derivatives and Scalar Differentiability 81
3.1.4 Characterization of Monotone Mappings by Using Scalar Derivatives 83
3.1.5 Computational Formulae for the Scalar Derivatives 86
3.2 Inversions 90
3.3 Fixed Point Theorems Generated by Krasnoselskii's Fixed Point Theorem 93
3.4 Surjectivity Theorems 94
3.5 Variational Inequalities and Complementarity Problems 97
3.6 Duality in Nonlinear Complementarity Theory 103
3.6.1 Preliminaries 104
3.6.2 Complementarity Problem 104
3.6.3 Exceptional Family of Elements 104
3.6.4 Infinitesimal Exceptional Family of Elements 106
3.6.5 A Duality and Main Results 107
3.7 Duality of Implicit Complementarity Problems 112
3.7.1 Implicit Complementarity Problem 112
3.7.2 Exceptional Family of Elements for an Ordered Pair of Mappings 113
3.7.3 Infinitesimal Exceptional Family of Elements for an Ordered Pair of Mappings 114
3.7.4 A Duality and Main Results 115
3.8 Duality of Multivalued Complementarity Problems 119
3.8.1 Preliminaries 120
3.8.2 Approachable and Approximable Mappings 121
3.8.3 Complementarity Problem 122
3.8.4 Inversions of Set-Valued Mappings 122
3.8.5 Exceptional Family of Elements 123
3.8.6 Infinitesimal Exceptional Family of Elements 125
3.8.7 A Duality and Main results 127
3.9 The Asymptotic Browder–Hartman–Stampacchia Condition and Interior Bands of ε-Solutions for Nonlinear Complementarity Problems 132

3.9.1 Preliminaries 134
3.9.2 The Browder–Hartman–Stampacchia Condition 135
3.9.3 The asymptotic Browder–Hartman–Stampacchia condition 138
3.9.4 Infinitesimal Interior-Point-ε-Exceptional Families 142
3.9.5 Results Related to Properties (a) and (b) of the Interior Band Mapping $\mathcal{U}$ 143
3.9.6 Comments 149
3.10 REFE-Acceptable Mappings and a Necessary and Sufficient Condition for the Nonexistence of Regular Exceptional Families of Elements 149
3.10.1 REFE-Acceptable Mappings 149
3.10.2 Mappings Without Regular Exceptional Family of Elements. A necessary and Sufficient Condition 157

4. Scalar Derivatives in Banach Spaces 161
4.1 Preliminaries 161
4.2 Semi-inner Products 162
4.3 Inversions 163
4.4 Scalar Derivatives 165
4.5 Fixed Point Theorems in Banach Spaces 166
4.5.1 A Fixed Point Index for α-condensing Mappings 166
4.5.2 An Altman-type Fixed Point Theorem 168
4.5.3 Integral Equations 171
4.5.4 Applications of Krasnoselskii-Type Fixed Point Theorems 172
4.5.5 Applications of Altman-Type Fixed Point Theorems 175

5. Monotone Vector Fields on Riemannian Manifolds and Scalar Derivatives 179
5.1 Geodesic Monotone Vector Fields 180
5.1.1 Geodesic Monotone Vector Fields and Convex Functionals 181
5.1.2 Geodesic Monotone Vector Fields and the First Variation of the Length of a Geodesic 182
5.1.3 Closed Geodesics and Geodesic Monotone Vector Fields 184
5.1.4 The Geodesic Monotonicity of Position Vector Fields 185
5.1.5 Geodesic Scalar Derivative 189
5.1.6 Geodesic Monotone Vector Fields on $\mathbb{S}^n$ 192
5.1.7 Geodesic Monotone Vector Fields on $\mathbb{H}^n$ 197

5.2 Killing Monotone Vector Fields 200
5.2.1 Expansive One-Parameter Transformation Groups 200
5.2.2 Geodesic Scalar Derivatives and Conformity 206
5.3 Projection Maps on Hadamard Manifolds 207
5.3.1 Some Basic Consequences of the Comparison Theorems 208
5.3.2 The Complementary Vector Field of a Map 214
5.3.3 Projection maps generating monotone vector fields 214
5.4 Nonexpansive Maps 219
5.4.1 Some Other Consequences of the Comparison Theorems 219
5.4.2 Nonexpansive Maps Generating Monotone Vector Fields 222
5.5 Zeros of Monotone Vector Fields 222
5.6 Homeomorphisms and Monotone Vector Fields 223
5.6.1 Preliminary Results 224
5.6.2 Homeomorphisms of Hadamard Manifolds 227
References 231
Index 241