Contents

Preface **v**

1 Two Examples **1**
1.1 The (Single) Nonlinear Center 1
1.2 The Nonsemisimple Double-Zero Eigenvalue 21

2 The Splitting Problem for Linear Operators **27**
2.1 The Splitting Problem in the Semisimple Case 28
2.2 Splitting by Means of an Inner Product 32
2.3 Nilpotent Operators 34
2.4 Canonical Forms 39
2.5 * An Introduction to sl(2) Representation Theory 47
2.6 * Algorithms for the sl(2) Splittings 57
2.7 * Obtaining sl(2) Triads 63

3 Linear Normal Forms **69**
3.1 Perturbations of Matrices 69
3.2 An Introduction to the Five Formats 74
3.3 Normal and Hypernormal Forms When A_0 Is Semisimple 87
3.4 Inner Product and Simplified Normal Forms 99
3.5 * The sl(2) Normal Form 118
3.6 Lie Theory and the Generated Formats 129
3.7 Metanormal Forms and k-Determined Hyperbolicity . . . 142

4 Nonlinear Normal Forms **157**
4.1 Preliminaries . . . 157
4.2 Settings for Nonlinear Normal Forms . . . 160
4.3 The Direct Formats (1a and 1b) . . . 164
4.4 Lie Theory and the Generated Formats . . . 174
4.5 The Semisimple Normal Form . . . 190
4.6 The Inner Product and Simplified Normal Forms . . . 221
4.7 The Module Structure of Inner Product and Simplified Normal Forms . . . 242
4.8 * The sl(2) Normal Form . . . 265
4.9 The Hamiltonian Case . . . 271
4.10 Hypernormal Forms for Vector Fields . . . 283

5 Geometrical Structures in Normal Forms **295**
5.1 Preserved Structures in Truncated Normal Forms . . . 297
5.2 Geometrical Structures in the Full System . . . 316
5.3 Error Estimates . . . 323
5.4 The Nilpotent Center Manifold Case . . . 335

6 Selected Topics in Local Bifurcation Theory **339**
6.1 Bifurcations from a Single-Zero Eigenvalue: A "Neoclassical" Approach . . . 341
6.2 Bifurcations from a Single-Zero Eigenvalue: A "Modern" Approach . . . 356
6.3 Unfolding the Single-Zero Eigenvalue . . . 366
6.4 Unfolding in the Presence of Generic Quadratic Terms . . . 371
6.5 Bifurcations from a Single Center (Hopf and Degenerate Hopf Bifurcations) . . . 382
6.6 Bifurcations from the Nonsemisimple Double-Zero Eigenvalue (Takens–Bogdanov Bifurcations) . . . 389
6.7 Unfoldings of Mode Interactions . . . 396

A Rings **405**
A.1 Rings, Ideals, and Division . . . 405
A.2 Monomials and Monomial Ideals . . . 412
A.3 Flat Functions and Formal Power Series . . . 421
A.4 Orderings of Monomials . . . 424
A.5 Division in Polynomial Rings; Gröbner Bases . . . 427
A.6 Division in Power Series Rings; Standard Bases . . . 438
A.7 Division in the Ring of Germs . . . 444

B Modules **447**
B.1 Submodules of $\mathbb{Z}^n$ . . . 447
B.2 Modules of Vector Fields . . . 449

C Format 2b: Generated Recursive (Hori) 451
C.1 Format 2b, Linear Case (for Chapter 3) 451
C.2 Format 2b, Nonlinear Case (for Chapter 4) 457

D Format 2c: Generated Recursive (Deprit) 463
D.1 Format 2c, Linear Case (for Chapter 3) 463
D.2 Format 2c, Nonlinear Case (for Chapter 4) 471

E On Some Algorithms in Linear Algebra 477

References 481

Index 489