Contents

Preface **v**

Introduction **1**

1 Fundamentals of the Optimal Experimental Design **5**
1.1 The Regression Equation 5
1.2 Gauss–Markov Theorem 6
1.3 Experimental Designs and Information Matrices 7
1.4 Optimality Criteria 9
1.4.1 D-Criterion 9
1.4.2 G-Criterion 10
1.4.3 MV-Criterion 10
1.4.4 c-Criterion 10
1.4.5 E-Criterion 11
1.5 Equivalence Theorems 11
1.6 Iterative Numerical Techniques 12
1.7 Nonlinear Regression Models 13
1.8 The Implicit Function Theorem 17
1.9 Chebyshev Models 18

2 The Functional Approach **23**
2.1 Introduction 23
2.2 Basic Ideas of the Functional Approach 25
2.2.1 Exponential regression models 25
2.2.2 Locally D-optimal designs 29
2.2.3 Maximin efficient designs 39
2.3 Description of the Model 44
2.3.1 Assumptions and notation 45
2.3.2 The basic equation 46
2.3.3 The uniqueness and the analytical properties 48
2.4 The Study of the Basic Equation 49
2.4.1 Properties of implicit functions 49
2.4.2 Jacobian of the basic equation 51
2.4.3 On the representation of implicit functions 53

2.4.4 The monotony property 55
2.5 Three-Parameter Logistic Distribution 60
2.6 Appendix: Proofs 63
2.6.1 Proof of Theorems 2.4.2, 2.4.3, and 2.4.4 63
2.6.2 Proof of Theorem 2.3.1 65
2.6.3 Proof of Theorem 2.2.3 67

3 Polynomial Models **71**
3.1 Introduction 71
3.2 Designs for Individual Coefficients 73
3.2.1 Statement of the problem 73
3.2.2 e_k-Optimal designs 74
3.2.3 Analytical properties of e_k-optimal designs 75
3.2.4 A numerical example 82
3.3 E-Optimal Designs: Preliminary Results 87
3.3.1 Statement of the problem and a dual theorem 88
3.3.2 The number of support points 89
3.3.3 Chebyshev designs 93
3.3.4 A boundary equation 103
3.4 Non-Chebyshev E-Optimal Designs 104
3.4.1 Basic equation 104
3.4.2 Limiting designs 114
3.4.3 Proof of the main theorem 121
3.4.4 Examples 124

4 Trigonometrical Models **135**
4.1 Introduction 135
4.2 D-Optimal Designs 137
4.2.1 Preliminary results for D-optimal designs 137
4.2.2 Analytic properties of D-optimal designs 142
4.2.3 The differential equation and the eigenvalue problem 148
4.2.4 A functional-algebraic approach 150
4.2.5 Examples 153
4.3 E-Optimal Designs 155
4.3.1 Preliminary results and E-optimal designs on large design spaces 155
4.3.2 E-optimal designs on sufficiently small intervals . . . 160
4.3.3 Example: The linear trigonometric regression model on a partial circle 172
4.3.4 E-Optimal designs on arbitrary intervals 174
4.4 Numerical Comparison of D- and E-Optimal Designs 192

5 D-Optimal Designs for Rational Models 197
5.1 Introduction . . . 197
5.2 Description of the Model . . . 198
5.3 The Number of Points . . . 200
5.4 Optimal Design Function . . . 203
5.5 Algebraic Approach and Limiting Designs . . . 205
5.6 The Taylor Expansion . . . 213

6 D-Optimal Designs for Exponential Models 217
6.1 The Number of Support Points . . . 218
6.2 Optimal Design Function . . . 224
6.3 Taylor Expansions . . . 228

7 E- and c-Optimal Designs 231
7.1 Introduction . . . 231
7.2 Preliminary Results . . . 234
7.3 Asymptotic Analysis of E- and c-Optimal Designs . . . 238
7.4 Analytical Properties of Optimal Designs . . . 243
7.5 Rational Models . . . 245
7.6 Exponential Models . . . 259
7.7 Appendix: Some Auxiliary Results . . . 268

8 The Monod Model 273
8.1 Introduction . . . 273
8.2 Equivalent Regression Models . . . 276
8.3 Locally D-Optimal Designs . . . 282
8.4 A Numerical Study . . . 285
8.5 Taylor Expansions . . . 289
8.6 Locally E- and e_k-Optimal Designs . . . 294
8.7 Maximin Efficient Designs . . . 298
8.7.1 A numerical procedure . . . 299
8.7.2 A comparison of maximin and uniform designs . . . 301
8.8 Appendix . . . 306
8.8.1 Proof of Lemma 8.2.1 . . . 310
8.8.2 Proof of Theorem 8.2.1 . . . 310
8.8.3 Proof of Lemma 8.3.1 . . . 312
8.8.4 Proof of Lemma 8.3.2 . . . 313
8.8.5 Proof of Lemma 8.6.1. . . . 315

Appendix 317

References 321

Index 331