

# Contents

<b>List of Tables .....</b>	xiii
<b>Notation and Conventions .....</b>	xv
<b>1 Polynomial Ideals and Their Varieties .....</b>	1
1.1 Fundamental Concepts .....	1
1.2 The Ideal Membership Problem and Gröbner Bases .....	7
1.3 Basic Properties and Algorithms .....	24
1.4 Decomposition of Varieties .....	38
1.5 Notes and Complements .....	50
Exercises .....	51
<b>2 Stability and Normal Forms .....</b>	57
2.1 Lyapunov's Second Method .....	57
2.2 Real Normal Forms .....	62
2.3 Analytic and Formal Normal Forms .....	68
2.4 Notes and Complements .....	83
Exercises .....	84
<b>3 The Center Problem .....</b>	89
3.1 The Poincaré First Return Map and the Lyapunov Numbers .....	91
3.2 Complexification of Real Systems, Normal Forms, and the Center Problem .....	96
3.3 The Center Variety .....	108
3.4 Focus Quantities and Their Properties .....	118
3.5 Hamiltonian and Reversible Systems .....	128
3.6 Darboux Integrals and Integrating Factors .....	136
3.7 Applications: Quadratic Systems and a Family of Cubic Systems .....	147
3.8 The Center Problem for Liénard Systems .....	158
3.9 Notes and Complements .....	164
Exercises .....	167

<b>4 The Isochronicity and Linearizability Problems</b> . . . . .	175
4.1 The Period Function . . . . .	175
4.2 Isochronicity Through Normal Forms and Linearizability . . . . .	177
4.3 The Linearizability Quantities . . . . .	191
4.4 Darboux Linearization . . . . .	199
4.5 Linearizable Quadratic Centers . . . . .	205
4.6 Notes and Complements . . . . .	208
Exercises . . . . .	209
<b>5 Invariants of the Rotation Group</b> . . . . .	213
5.1 Properties of Invariants . . . . .	214
5.2 The Symmetry Ideal and the Set of Time-Reversible Systems . . . . .	229
5.3 Axes of Symmetry of a Plane System . . . . .	237
5.4 Notes and Complements . . . . .	244
Exercises . . . . .	245
<b>6 Bifurcations of Limit Cycles and Critical Periods</b> . . . . .	249
6.1 Bautin's Method for Bifurcation Problems . . . . .	250
6.2 The Cyclicity Problem . . . . .	257
6.3 The Cyclicity of Quadratic Systems and a Family of Cubic Systems	269
6.4 Bifurcations of Critical Periods . . . . .	287
6.5 Notes and Complements . . . . .	299
Exercises . . . . .	301
<b>Appendix</b> . . . . .	307
<b>References</b> . . . . .	313
<b>Index of Notation</b> . . . . .	323
<b>Index</b> . . . . .	327