

---

# Contents

Preface .....	VII
---------------	-----

---

## Part I The locus of a perturbed relay system theory

---

<b>1 The servo problem in discontinuous control systems .....</b>	<b>3</b>
1.1 Introduction .....	3
1.2 Fundamentals of frequency-domain analysis of periodic motions in nonlinear systems.....	5
1.3 Relay servo systems.....	10
1.4 Symmetric oscillations in relay servo systems: DF analysis ....	12
1.5 Asymmetric oscillations in relay servo systems: DF analysis ...	14
1.6 Slow signal propagation through a relay servo system .....	16
1.7 Conclusions.....	17
<b>2 The locus of a perturbed relay system (LPRS) theory .....</b>	<b>19</b>
2.1 Introduction to the LPRS .....	19
2.2 Computing the LPRS for a non-integrating plant .....	21
2.2.1 Matrix state-space description approach .....	21
2.2.2 Partial fraction expansion technique .....	23
2.2.3 Transfer function description approach .....	24
2.2.4 Orbital stability of relay systems .....	26
2.3 Computing the LPRS for an integrating plant.....	26
2.3.1 Matrix state-space description approach .....	26
2.3.2 Transfer function description approach .....	29
2.3.3 Orbital stability of relay systems .....	30
2.4 Computing the LPRS for a plant with a time delay .....	31
2.4.1 Matrix state-space description approach .....	31
2.4.2 Orbital asymptotic stability .....	32
2.5 LPRS of first-order dynamics .....	33
2.6 LPRS of second-order dynamics .....	35

2.7	LPRS of first-order plus dead-time dynamics . . . . .	38
2.8	Some properties of the LPRS . . . . .	41
2.9	LPRS of nonlinear plants . . . . .	43
2.9.1	Additivity property . . . . .	43
2.9.2	The LPRS extended definition and open-loop LPRS computing . . . . .	46
2.10	Application of periodic signal mapping to computing the LPRS of some special nonlinear plants . . . . .	48
2.11	Comparison of the LPRS with other methods of analysis of relay systems . . . . .	52
2.12	An example of analysis of oscillations and transfer properties . .	53
2.13	Conclusions . . . . .	54
<b>3</b>	<b>Input-output analysis of relay servo systems . . . . .</b>	<b>57</b>
3.1	Slow and fast signal propagation through a relay servo system . . . . .	57
3.2	Methodology of input-output analysis . . . . .	63
3.3	Example of forced motions analysis with the use of the LPRS .	63
3.4	Conclusions . . . . .	65
<b>4</b>	<b>Analysis of sliding modes in the frequency domain . . . . .</b>	<b>67</b>
4.1	Introduction to sliding mode control . . . . .	67
4.2	Representation of a sliding mode system via the equivalent relay system . . . . .	69
4.3	Analysis of motions in the equivalent relay system . . . . .	73
4.4	The chattering phenomenon and its LPRS analysis . . . . .	77
4.5	Reduced-order and non-reduced-order models of averaged motions in a sliding mode system and input-output analysis . .	85
4.6	On fractal dynamics in sliding-mode control . . . . .	88
4.7	Examples of chattering and disturbance attenuation analysis . .	95
4.8	Conclusions . . . . .	101
<b>5</b>	<b>Performance analysis of second-order SM control algorithms . . . . .</b>	<b>103</b>
5.1	Introduction . . . . .	103
5.2	Sub-optimal algorithm . . . . .	104
5.3	Describing function analysis of chattering . . . . .	105
5.4	Exact frequency-domain analysis of chattering . . . . .	106
5.5	Describing function analysis of external signal propagation . . .	108
5.6	Exact frequency-domain analysis of external signal propagation . . . . .	112
5.7	Example of the analysis of sub-optimal algorithm performance . . . . .	117
5.8	Conclusions . . . . .	122

---

**Part II Applications of the locus of a perturbed relay system**

---

<b>6</b>	<b>Relay pneumatic servomechanism design</b> .....	125
6.1	Relay pneumatic servomechanism dynamics and characteristics .....	125
6.2	LPRS analysis of uncompensated relay electro-pneumatic servomechanism .....	127
6.3	Compensator design in the relay electro-pneumatic servomechanism .....	128
6.4	Examples of compensator design in the relay electro-pneumatic servomechanism .....	132
6.5	Compensator design in the relay electro-pneumatic servomechanism with the use of the LPRS of a nonlinear plant .....	135
6.6	Conclusions .....	138
<b>7</b>	<b>Relay feedback test identification and autotuning</b> .....	139
7.1	The relay feedback test .....	139
7.2	The LPRS and asymmetric relay feedback test .....	140
7.3	Methodology of identification of the first-order plus dead-time process .....	141
7.4	Analysis of potential sources of inaccuracy .....	143
7.5	Performance analysis of the identification algorithm .....	145
7.6	Tuning algorithm .....	147
7.7	Conclusions .....	151
<b>8</b>	<b>Performance analysis of the sliding mode-based analog differentiator and dynamical compensator</b> .....	153
8.1	Transfer function “inversion” via sliding mode .....	153
8.2	Analysis of SM differentiator dynamics .....	154
8.3	Temperature sensor dynamics compensation via SM application .....	157
8.4	Analysis of the sliding mode compensator .....	160
8.5	An example of compensator design .....	162
8.6	Conclusions .....	165
<b>9</b>	<b>Analysis of sliding mode observers</b> .....	167
9.1	The SM observer as a relay servo system .....	167
9.2	SM observer performance analysis and characteristics .....	170
9.3	Example of SM observer performance analysis .....	172
9.4	Conclusions .....	175

<b>10 Appendix</b> .....	177
10.1 The LPRS derivation for a non-integrating linear part .....	177
10.2 Orbital stability of a system with a non-integrating linear part	181
10.3 The LPRS derivation for an integrating linear part .....	183
10.4 Orbital stability of a system with an integrating linear part ...	191
10.5 The LPRS derivation for a linear part with time delay .....	194
10.6 MATLAB code for LPRS computing .....	198
<b>References</b> .....	205
<b>Index</b> .....	211