

Table of Contents

Introduction	1
<hr/>	
I The Hahn-Banach-Lagrange theorem and some consequences	
<hr/>	
1 The Hahn–Banach–Lagrange theorem	15
2 Applications to functional analysis	23
3 A minimax theorem	24
4 The dual and bidual of a normed space	25
5 Excess, duality gap, and minimax criteria for weak compactness	28
6 Sharp Lagrange multiplier and KKT results	32
<hr/>	
II Fenchel duality	
<hr/>	
7 A sharp version of the Fenchel Duality theorem	41
8 Fenchel duality with respect to a bilinear form — locally convex spaces	44
9 Some properties of $\frac{1}{2}\ \cdot\ ^2$	49
10 The conjugate of a sum in the locally convex case	51
11 Fenchel duality vs the conjugate of a sum	54
12 The restricted biconjugate and Fenchel–Moreau points	58
13 Surrounding sets and the dom lemma	60
14 The \ominus-theorem	62
15 The Attouch–Brezis theorem	65
16 A bivariate Attouch–Brezis theorem	67

XII Table of Contents

III Multifunctions, SSD spaces, monotonicity and Fitzpatrick functions

17	Multifunctions, monotonicity and maximality	71
18	Subdifferentials are maximally monotone	74
19	SSD spaces, q -positive sets and BC-functions	79
20	Maximally q -positive sets in SSD spaces	86
21	SSDB spaces	88
22	The SSD space $E \times E^*$	93
23	Fitzpatrick functions and fitzpatrifications	99
24	The maximal monotonicity of a sum	103

IV Monotone multifunctions on general Banach spaces

25	Monotone multifunctions with bounded range	107
26	A general local boundedness theorem	108
27	The six set theorem and the nine set theorem	108
28	$D(S_\varphi)$ and various hulls	111

V Monotone multifunctions on reflexive Banach spaces

29	Criteria for maximality, and Rockafellar's surjectivity theorem	117
30	Surjectivity and an abstract Hammerstein theorem	123
31	The Brezis–Haraux condition	125
32	Bootstrapping the sum theorem	128
33	The $>$ six set and the $>$ nine set theorems for pairs of multifunctions	130
34	The Brezis–Crandall–Pazy condition	132

VI Special maximally monotone multifunctions

35	The norm-dual of the space $E \times E^*$ and $\widetilde{\text{BC}}$ -functions	139
36	Subclasses of the maximally monotone multifunctions	147
37	First application of Theorem 35.8: type (D) implies type (FP)	153
38	$\mathcal{T}_{\mathcal{CLB}}(E^{**})$, $\mathcal{T}_{\mathcal{CLBN}}(B^*)$ and type (ED)	154
39	Second application of Theorem 35.8: type (ED) implies type (FPV)	157
40	Final applications of Theorem 35.8: type (ED) implies strong	158
41	Strong maximality and coercivity	159
42	Type (ED) implies type (ANA) and type (BR)	161
43	The closure of the range	167
44	The sum problem and the closure of the domain	170
45	The biconjugate of a maximum and $\mathcal{T}_{\mathcal{CLB}}(E^{**})$	172
46	Maximally monotone multifunctions with convex graph	180
47	Possibly discontinuous positive linear operators	183
48	Subtler properties of subdifferentials	188
49	Saddle functions and type (ED)	192

VII The sum problem for general Banach spaces

50	Introductory comments	197
51	Voisei's theorem	197
52	Sums with normality maps	198
53	A theorem of Verona–Verona	199

XIV Table of Contents

VIII Open problems	203
<hr/>	<hr/>
IX Glossary of classes of multifunctions	205
<hr/>	<hr/>
X A selection of results	207
<hr/>	<hr/>
References	233
Subject index	239
Symbol index	243