

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	The Basic Idea	1
1.2	The Design Process	1
1.3	General Mathematical Form of a Structural Optimization Problem	3
1.4	Three Types of Structural Optimization Problems	5
1.5	Discrete and Distributed Parameter Systems	7
<b>2</b>	<b>Examples of Optimization of Discrete Parameter Systems</b>	9
2.1	Weight Minimization of a Two-Bar Truss Subject to Stress Constraints	9
2.2	Weight Minimization of a Two-Bar Truss Subject to Stress and Instability Constraints	12
2.3	Weight Minimization of a Two-Bar Truss Subject to Stress and Displacement Constraints	14
2.4	Weight Minimization of a Two-Beam Cantilever Subject to a Displacement Constraint	18
2.5	Weight Minimization of a Three-Bar Truss Subject to Stress Constraints	21
2.6	Weight Minimization of a Three-Bar Truss Subject to a Stiffness Constraint	31
2.7	Exercises	33
<b>3</b>	<b>Basics of Convex Programming</b>	35
3.1	Local and Global Optima	35
3.2	Convexity	37
3.3	KKT Conditions	41
3.4	Lagrangian Duality	46
3.4.1	Lagrangian Duality for Convex and Separable Problems	47
3.5	Exercises	52
<b>4</b>	<b>Sequential Explicit, Convex Approximations</b>	57
4.1	General Solution Procedure for Nested Problems	57

4.2	Sequential Linear Programming (SLP)	58
4.3	Sequential Quadratic Programming (SQP)	59
4.4	Convex Linearization (CONLIN)	59
4.5	The Method of Moving Asymptotes (MMA)	66
4.6	Exercises	72
<b>5</b>	<b>Sizing Stiffness Optimization of a Truss</b>	<b>77</b>
5.1	The Simultaneous Formulation of the Problem	77
5.2	The Nested Formulation and Some of Its Properties	84
5.2.1	Convexity of the Nested Problem	85
5.2.2	Fully Stressed Designs	87
5.2.3	Minimization of the Volume Under a Compliance Constraint	88
5.3	Numerical Solution of the Nested Problem Using MMA	91
<b>6</b>	<b>Sensitivity Analysis</b>	<b>97</b>
6.1	Numerical Methods	97
6.2	Analytical Methods	98
6.2.1	Direct Analytical Method	98
6.2.2	Adjoint Analytical Method	99
6.3	Analytical Calculation of Pseudo-loads	100
6.3.1	Bars	101
6.3.2	Plane Sheets	104
6.4	Exercises	112
<b>7</b>	<b>Two-Dimensional Shape Optimization</b>	<b>117</b>
7.1	Shape Representation	117
7.1.1	Bézier Splines	118
7.1.2	B-Splines	120
7.2	Treatment of Geometrical Design Constraints	127
7.2.1	$C^1$ Continuity Between Bézier Splines	128
7.2.2	$C^1$ Continuity at a Point on a Line of Symmetry	129
7.2.3	A Composite Circular Arc	131
7.3	Mesh Generation and Calculation of Nodal Sensitivities	132
7.3.1	B-Spline Surface Meshes	133
7.3.2	Coons Surface Meshes	134
7.3.3	Unstructured Meshes	137
7.4	Summary of Sensitivity Analysis for Two-Dimensional Shape Optimization	139
7.5	Exercises	143
<b>8</b>	<b>Stiffness Optimization of Distributed Parameter Systems</b>	<b>147</b>
8.1	Calculus of Variations	147
8.1.1	Optimality Conditions and Gateaux Derivatives	149
8.1.2	Handling a Constraint	153
8.2	Equilibrium Principles for Distributed Parameter Systems	156
8.2.1	One-Dimensional Elasticity	156

- 8.2.2 Beam Problem . . . . . 158
- 8.2.3 Two-Dimensional Elasticity . . . . . 159
- 8.2.4 Abstract Equilibrium Principles . . . . . 162
- 8.3 The Design Problem . . . . . 163
  - 8.3.1 Optimality Conditions . . . . . 166
  - 8.3.2 The Stiffest Rod . . . . . 168
  - 8.3.3 Beam Stiffness Optimization . . . . . 170
- 8.4 Exercises . . . . . 174
- 9 Topology Optimization of Distributed Parameter Systems . . . . . 179**
  - 9.1 The Variable Thickness Sheet Problem . . . . . 179
    - 9.1.1 Problem Statement and FE-Discretization . . . . . 179
    - 9.1.2 The Optimality Criteria (OC) Method . . . . . 182
  - 9.2 Penalization of Intermediate Thickness Values . . . . . 188
    - 9.2.1 Solid Isotropic Material with Penalization (SIMP) . . . . . 188
    - 9.2.2 Other Penalizations . . . . . 190
  - 9.3 Well-Posedness and Potential Numerical Problems . . . . . 190
    - 9.3.1 The Archetype Problem and an Analogy . . . . . 190
    - 9.3.2 Numerical Instabilities . . . . . 191
  - 9.4 Restriction of the Archetype Problem . . . . . 193
    - 9.4.1 Bounds on the Design Gradient . . . . . 194
    - 9.4.2 Filters . . . . . 195
  - 9.5 Relaxation of the Archetype Problem . . . . . 198
  - 9.6 Exercises . . . . . 200
- Answers to Selected Exercises . . . . . 203**
- References . . . . . 207**
- Index . . . . . 209**