
Contents

1	Introduction	1
1.1	Solution Methods for Wave Propagation Problems	1
1.2	Fourier Analysis	6
1.2.1	Continuous Fourier Transforms	6
1.2.2	Fourier Series	9
1.2.3	Discrete Fourier Transform	11
1.3	Spectral Analysis	15
1.4	What is the Spectral Element Method?	19
1.5	Outline and Scope of Book	21
2	Introduction to the Theory of Anisotropic and Inhomogeneous Materials	23
2.1	Introduction to Composite Materials	23
2.2	Theory of Laminated Composites	24
2.2.1	Micromechanical Analysis of a Lamina	25
2.2.2	Strength of Materials Approach to Determination of Elastic Moduli	25
2.2.3	Stress–Strain Relations for a Lamina	29
2.2.4	Stress–Strain Relation for a Lamina with Arbitrary Orientation of Fibers	31
2.3	Introduction to Smart Composites	34
2.4	Modeling Inhomogeneous Materials	38
3	Idealization of Wave Propagation and Solution Techniques .	41
3.1	General Form of the Wave Equations	41
3.2	Characteristics of Waves in Anisotropic Media	42
3.3	General Form of Inhomogeneous Wave Equations	43
3.4	Basic Properties and Solution Techniques	43
3.5	Spectral Finite Element Discretization	44
3.6	Efficient Computation of the Wavenumber and Wave Amplitude	48

3.6.1	Method 1: The Companion Matrix and the SVD Technique	49
3.6.2	Method 2: Linearization of PEP	50
3.7	Spectral Element Formulation for Isotropic Material	51
3.7.1	Spectral Element for Rods	51
3.7.2	Spectral Element for Beams	53
4	Wave Propagation in One-dimensional Anisotropic Structures	55
4.1	Wave Propagation in Laminated Composite Thin Rods and Beams	55
4.1.1	Governing Equations and PEP	56
4.1.2	Spectrum and Dispersion Relations	58
4.2	Spectral Element Formulation	59
4.2.1	Finite Length Element	59
4.2.2	Throw-off Element	61
4.3	Numerical Results and Discussions	61
4.3.1	Impact on a Cantilever Beam	61
4.3.2	Effect of the Axial–Flexural Coupling	63
4.3.3	Wave Transmission and Scattering Through an Angle-joint	66
4.4	Wave Propagation in Laminated Composite Thick Beams: Poisson’s Contraction and Shear Deformation Models	69
4.4.1	Wave Motion in a Thick Composite Beam	70
4.4.2	Coupled Axial–Flexural Shear and Thickness Contractional Modes	72
4.4.3	Correction Factors at High Frequency Limit	74
4.4.4	Coupled Axial–Flexural Shear Without the Thickness Contractional Modes	76
4.4.5	Modeling Spatially Distributed Dynamic Loads	79
4.5	Modeling Damping Using Spectral Element	81
4.5.1	Proportional Damping Through a Discretized Finite Element Model	81
4.5.2	Proportional Damping Through the Wave Equation ...	83
4.6	Numerical Results and Discussions	88
4.6.1	Comparison of Response with Standard FEM	91
4.6.2	Presence of Axial–Flexural Shear Coupling	93
4.6.3	Parametric Studies on a Cantilever Beam	96
4.6.4	Response of a Beam with Ply-drops	96
4.7	Layered Composite Thin-walled Tubes	99
4.7.1	Linear Wave Motion in Composite Tube	102
4.8	Spectral Finite Element Model	107
4.8.1	Short and Long Wavelength Limits for Thin Shell and Limitations of the Proposed Model	107
4.8.2	Comparison with Analytical Solution	114

4.9	Numerical Simulations	116
4.9.1	Time Response Under Short Impulse Load and the Effect of Fiber Orientations	116
5	Wave Propagation in One-dimensional Inhomogeneous Structures	123
5.1	Length-wise Functionally Graded Rod	124
5.1.1	Development of Spectral Finite Elements	126
5.1.2	Smoothing of Reflected Pulse	132
5.2	Depth-wise Functionally Graded Beam	135
5.2.1	Spectral Finite Element Formulation	137
5.2.2	The Spectrum and Dispersion Relation	137
5.2.3	Effect of Gradation on the Cut-off Frequencies	139
5.2.4	Computation of the Temperature Field	142
5.3	Wave Propagation Analysis: Depth-wise Graded Beam (HMT)	142
5.3.1	Validation of the Formulated SFE	143
5.3.2	Lamb Wave Propagation in FSDT and HMT Beams ..	148
5.3.3	Effect of Gradation on Stress Waves	151
5.3.4	Coupled Thermoelastic Wave Propagation	153
5.4	Length-wise Graded Beam: FSDT	157
5.4.1	Spectral Finite Element Formulation	158
5.4.2	Effect of Gradation on the Spectrum and Dispersion Relation	159
5.4.3	Effect of Gradation on the Cut-off Frequencies	160
5.5	Numerical Examples	162
5.5.1	Effect of the Inhomogeneity	162
5.5.2	Elimination of the Reflection from Material Boundary ..	165
6	Wave Propagation in Two-dimensional Anisotropic Structures	171
6.1	Two-dimensional Initial Boundary Value Problem	172
6.2	Spectral Element for Doubly Bounded Media	176
6.2.1	Finite Layer Element (FLE)	177
6.2.2	Infinite Layer Element (ILE)	178
6.2.3	Expressions for Stresses and Strains	178
6.2.4	Prescription of Boundary Conditions	179
6.2.5	Determination of Lamb Wave Modes	179
6.3	Numerical Examples	181
6.3.1	Propagation of Surface and Interface Waves	181
6.3.2	Propagation of Lamb Wave	185
7	Wave Propagation in Two-dimensional Inhomogeneous Structures	195
7.1	SLE Formulation: Inhomogeneous Media	195
7.1.1	Exact Formulation	196

- 7.2 Numerical Examples 201
 - 7.2.1 Propagation of Stress Waves 201
 - 7.2.2 Propagation of Lamb Waves 204
- 7.3 SLE Formulation: Thermoelastic Analysis 208
 - 7.3.1 Inhomogeneous Anisotropic Material 209
 - 7.3.2 Discussion on the Properties of Wavenumbers 212
 - 7.3.3 Finite Layer Element (FLE) 215
 - 7.3.4 Infinite Layer Element (ILE) 216
 - 7.3.5 Homogeneous Anisotropic Material 217
- 7.4 Numerical Examples 217
 - 7.4.1 Effect of the Relaxation Parameters - Symmetric Ply-layup 217
 - 7.4.2 Interfacial Waves: Thermal and Mechanical Loading ... 220
 - 7.4.3 Propagation of Stress Waves 221
 - 7.4.4 Propagation of Thermal Waves 226
 - 7.4.5 Effect of Inhomogeneity 227
- 7.5 Wave Motion in Anisotropic and Inhomogeneous Plate 229
 - 7.5.1 SPE Formulation: CLPT 230
 - 7.5.2 Computation of Wavenumber: Anisotropic Plate 234
 - 7.5.3 Computation of Wavenumber: Inhomogeneous Plate ... 237
 - 7.5.4 The Finite Plate Element 241
 - 7.5.5 Semi-infinite or Throw-off Plate Element 242
- 7.6 Numerical Examples 243
 - 7.6.1 Wave Propagation in Plate with Ply-drop 243
 - 7.6.2 Propagation of Lamb waves 246
- 8 Solution of Inverse Problems: Source and System Identification 249**
 - 8.1 Force Identification 249
 - 8.1.1 Force Reconstruction from Truncated Response 250
 - 8.2 Material Property Identification 253
 - 8.2.1 Estimation of Material Properties: Inhomogeneous Layer 254
- 9 Application of SFEM to SHM: Simplified Damage Models . 259**
 - 9.1 Various Damage Identification Techniques 259
 - 9.1.1 Techniques for Modeling Delamination 260
 - 9.1.2 Modeling Issues in Structural Health Monitoring 261
 - 9.2 Modeling Wave Scattering due to Multiple Delaminations and Inclusions 262
 - 9.3 Spectral Element with Embedded Delamination 265
 - 9.3.1 Modeling Distributed Contact Between Delaminated Surfaces 269
 - 9.4 Numerical Studies on Wave Scattering due to Single Delamination 271
 - 9.4.1 Comparison with 2-D FEM 271

9.4.2 Identification of Delamination Location from Scattered Wave 273

9.4.3 Effect of Delamination at Ply-drops 274

9.4.4 Sensitivity of the Delaminated Configuration 276

9.5 A Sublaminar-wise Constant Shear Kinematics Model 279

9.6 Spectral Elements with Embedded Transverse Crack 284

9.6.1 Element-internal Discretization and Kinematic Assumptions 284

9.6.2 Modeling Dynamic Contact Between Crack Surfaces ... 288

9.6.3 Modeling Surface-breaking Cracks 290

9.6.4 Distributed Constraints at the Interfaces Between Sublaminates and Hanging Laminates 291

9.7 Numerical Simulations 293

9.7.1 Comparison with 2-D FEM 293

9.7.2 Identification of Crack Location from Scattered Wave .. 294

9.7.3 Sensitivity of the Crack Configuration 296

9.8 Spectral Finite Element Model for Damage Estimation 297

9.8.1 Spectral Element with Embedded Degraded Zone 300

9.9 Numerical Simulations 301

10 Application of SFEM to SHM: Efficient Damage

Detection Techniques 307

10.1 Strategies for Identification of Damage in Composites 307

10.2 Spectral Power Flow 311

10.2.1 Properties of Spectral Power 312

10.2.2 Measurement of Wave Scattering due to Delaminations and Inclusions Using Spectral Power 314

10.3 Power Flow Studies on Wave Scattering 314

10.3.1 Wave Scattering due to Single Delamination 314

10.3.2 Wave Scattering due to Length-wise Multiple Delaminations 316

10.3.3 Wave Scattering due to Depth-wise Multiple Delaminations 317

10.4 Wave Scattering due to Strip Inclusion 319

10.4.1 Power Flow in a Semi-infinite Strip Inclusion with Bounded Media: Effect of Change in the Material Properties 319

10.4.2 Effect of Change in the Material Properties of a Strip Inclusion 321

10.5 Damage Force Indicator for SFEM 323

10.6 Numerical Simulation of Global Identification Process 327

10.6.1 Effect of Single Delamination 327

10.6.2 Effect of Multiple Delaminations 329

10.6.3 Sensitivity of Damage Force Indicator due to Variation in Delamination Size 330

10.6.4	Sensitivity of Damage Force Indicator due to Variation in Delamination Depth	331
10.7	Genetic Algorithm (GA) for Delamination Identification	337
10.7.1	Objective Functions in GA for Delamination Identification	338
10.7.2	Displacement-based Objective Functions	338
10.7.3	Power-based Objective Functions	343
10.8	Case Studies with a Cantilever Beam	346
10.8.1	Identification of Delamination Location	346
10.8.2	Identification of Delamination Size	348
10.8.3	Identification of Delamination Location and Size	349
10.8.4	Identification of Delamination Location, Size and Depth	349
10.8.5	Effect of Delamination Near the Boundary	350
10.9	Neural Network Integrated with SFEM	352
10.10	Numerical Results and Discussion	357
11	Spectral Finite Element Method for Active Wave Control	365
11.1	Challenges in Designing Active Broadband Control Systems	365
11.1.1	Strategies for Vibration and Wave Control	366
11.1.2	Active LAC of Structural Waves	371
11.2	Externally Mounted Passive/Active Devices	372
11.3	Modeling Distributed Transducer Devices	377
11.3.1	Plane Stress Constitutive Model of Stacked and Layered Piezoelectric Composite	378
11.3.2	Constitutive Model for Piezoelectric Fiber Composite (PFC)	381
11.3.3	Design Steps for Broadband Control	391
11.4	Active Spectral Finite Element Model	394
11.4.1	Spectral Element for Finite Beams	394
11.4.2	Sensor Element	395
11.4.3	Actuator Element	395
11.4.4	Numerical Implementation	397
11.5	Effect of Broadband Distributed Actuator Dynamics	398
11.6	Active Control of Multiple Waves in Helicopter Gearbox Support Struts	402
11.6.1	Active Strut System	404
11.6.2	Numerical Simulations	405
11.7	Optimal Control Based on ASFEM and Power Flow	415
11.7.1	Linear Quadratic Optimal Control Using Spectral Power	416
11.7.2	Broadband Control of a Three-member Composite Beam Network	417
	References	423
	Index	439