

Contents

1	Some Geometric Properties of Banach Spaces	1
1.1	Introduction	1
1.2	Uniformly Convex Spaces	2
1.3	Strictly Convex Banach Spaces	3
1.4	The Modulus of Convexity	4
1.5	Uniform Convexity, Strict Convexity and Reflexivity	7
1.6	Historical Remarks	9
2	Smooth Spaces	11
2.1	Introduction	11
2.2	The Modulus of Smoothness	11
2.3	Duality Between Spaces	14
2.4	Historical Remarks	18
3	Duality Maps in Banach Spaces	19
3.1	Motivation	19
3.2	Duality Maps of Some Concrete Spaces	23
3.3	Historical Remarks	28
4	Inequalities in Uniformly Convex Spaces	29
4.1	Introduction	29
4.2	Basic Notions of Convex Analysis	30
4.3	p -uniformly Convex Spaces	34
4.4	Uniformly Convex Spaces	39
4.5	Historical Remarks	44
5	Inequalities in Uniformly Smooth Spaces	45
5.1	Definitions and Basic Theorems	45
5.2	q -uniformly Smooth Spaces	46
5.3	Uniformly Smooth Spaces	49

5.4	Characterization of Some Real Banach Spaces by the Duality Map	51
5.4.1	Duality Maps on Uniformly Smooth spaces	51
5.4.2	Duality Maps on Spaces with Uniformly Gâteaux Differentiable Norms	51
6	Iterative Method for Fixed Points of Nonexpansive Mappings	57
6.1	Introduction	57
6.2	Asymptotic Regularity	59
6.3	Uniform Asymptotic Regularity	61
6.4	Strong Convergence	66
6.5	Weak Convergence	68
6.6	Some Examples	69
6.7	Halpern-type Iteration Method	73
6.7.1	Convergence Theorems	77
6.7.2	The Case of Non-self Mappings	80
6.8	Historical Remarks	84
7	Hybrid Steepest Descent Method for Variational Inequalities	87
7.1	Introduction	87
7.2	Preliminaries	91
7.3	Convergence Theorems	92
7.4	Further Convergence Theorems	99
7.4.1	Convergence Theorems	99
7.5	The Case of L_p spaces, $1 < p \leq 2$	107
7.6	Historical Remarks	111
8	Iterative Methods for Zeros of Φ-Accretive-Type Operators	113
8.1	Introduction and Preliminaries	113
8.2	Some Remarks on Accretive Operators	116
8.3	Lipschitz Strongly Accretive Maps	117
8.4	Generalized Phi-Accretive Self-Maps	120
8.5	Generalized Phi-Accretive Non-self Maps	124
8.6	Historical Remarks	127
9	Iteration Processes for Zeros of Generalized Φ-Accretive Mappings	129
9.1	Introduction	129
9.2	Uniformly Continuous Generalized Φ -hemi-contractive Maps .	130
9.3	Generalized Lipschitz, Generalized Φ -quasi-accretive Mappings	134
9.4	Historical Remarks	138

10 An Example; Mann Iteration for Strictly Pseudo-contractive Mappings	141
10.1 Introduction and a Convergence Theorem	141
10.2 An Example	141
10.3 Mann Iteration for a Class of Lipschitz Pseudo-contractive Maps	145
10.4 Historical Remarks	149
11 Approximation of Fixed Points of Lipschitz Pseudo-contractive Mappings	151
11.1 Iteration Methods for Lipschitz Pseudo-contractions	151
11.2 Historical Remarks	157
12 Generalized Lipschitz Accretive and Pseudo-contractive Mappings	161
12.1 Introduction	161
12.2 Convergence Theorems	161
12.3 Some Applications	166
12.4 Historical Remarks	166
13 Applications to Hammerstein Integral Equations	169
13.1 Introduction	169
13.2 Solution of Hammerstein Equations	169
13.2.1 Convergence Theorems for Lipschitz Maps	175
13.2.2 Convergence Theorems for Bounded Maps	177
13.2.3 Explicit Algorithms	178
13.3 Convergence Theorems with Explicit Algorithms	179
13.3.1 Some Useful Lemmas	179
13.3.2 Convergence Theorems with Coupled Schemes for the Case of Lipschitz Maps	180
13.3.3 Convergence in L_p , $1 < p \leq 2$	183
13.4 Coupled Scheme for the Case of Bounded Operators	185
13.4.1 Convergence Theorems	185
13.4.2 Convergence for Bounded Operators in L_p Spaces, $1 < p \leq 2$	188
13.4.3 Convergence Theorems for Generalized Lipschitz Maps	188
13.5 Remarks and Open Questions	190
13.6 Historical Remarks	191
14 Iterative Methods for Some Generalizations of Nonexpansive Maps	193
14.1 Introduction	193
14.2 Iteration Methods for Asymptotically Nonexpansive Mappings	193
14.2.1 Modified Mann Process	193

14.2.2 Iteration Method of Schu	197
14.2.3 Halpern-type Process	198
14.3 Asymptotically Quasi-nonexpansive Mappings	200
14.4 Historical Remarks	204
15 Common Fixed Points for Finite Families of Nonexpansive Mappings	205
15.1 Introduction	205
15.2 Convergence Theorems for a Family of Nonexpansive Mappings	209
15.3 Non-self Mappings	213
16 Common Fixed Points for Countable Families of Nonexpansive Mappings	215
16.1 Introduction	215
16.2 Path Convergence Theorems	218
16.3 Path Convergence in Uniformly Convex Real Banach Spaces .	220
16.4 Iterative Convergence in Uniformly Convex Real Banach Spaces	223
16.5 Non-self Mappings	227
16.6 Historical Remarks	229
17 Common Fixed Points for Families of Commuting Nonexpansive Mappings	231
17.1 Introduction	231
17.2 Three Commuting Nonexpansive Mappings	232
17.3 Common Fixed Points for Family of Commuting Nonexpansive Mappings	237
17.4 Convergence Theorems for Infinite Family of Commuting Nonexpansive Mappings	239
17.5 Historical Remarks	242
18 Finite Families of Lipschitz Pseudo-contractive and Accretive Mappings	243
18.1 Introduction	243
18.2 Convergence Theorems	243
18.3 Finite Families of Lipschitz Accretive Operators	249
18.4 Some Applications	250
18.5 Historical Remarks	250
19 Generalized Lipschitz Pseudo-contractive and Accretive Mappings	251
19.1 Introduction	251
19.2 Generalized Lipschitz Pseudo-contractive Mappings	251
19.3 Generalized Lipschitz Accretive Operators	255
19.4 Some Applications	255
19.5 Historical Remarks	256

20 Finite Families of Non-self Asymptotically Nonexpansive Mappings	257
20.1 Introduction	257
20.2 Preliminaries.....	259
20.3 Strong Convergence Theorems	260
20.4 Weak Convergence Theorems	267
20.5 The Case for Nonexpansive Mappings	269
20.6 Historical Remarks	270
21 Families of Total Asymptotically Nonexpansive Maps	271
21.1 Introduction	271
21.2 Convergence Theorems	273
21.2.1 Necessary and Sufficient Conditions for Convergence in Real Banach Spaces	275
21.2.2 Convergence Theorem in Real Uniformly Convex Banach Spaces	277
21.3 The Case of Non-self Maps	282
21.4 Historical Remarks	282
22 Common Fixed Points for One-parameter Nonexpansive Semigroup	283
22.1 Introduction	283
22.2 Existence Theorems	284
22.3 Convergence Theorems	285
22.4 Historical Remarks	285
23 Single-valued Accretive Operators; Applications; Some Open Questions	287
23.1 Introduction	287
23.2 Lower Semi-continuous Accretive Operators are Single-valued	287
23.3 An Application to Variational Inequalities	293
23.4 General Comments on Some Fixed Point Theorems	295
23.5 Examples of Accretive Operators	296
23.6 Examples of Nonexpansive Retracts.....	297
23.7 Some Questions of Interest	297
23.8 Further Reading.....	299
References	301
Index	325