Series Editors' Foreword

The topics of control engineering and signal processing continue to flourish and develop. In common with general scientific investigation, new ideas, concepts and interpretations emerge quite spontaneously and these are then discussed, used, discarded or subsumed into the prevailing subject paradigm. Sometimes these innovative concepts coalesce into a new sub-discipline within the broad subject tapestry of control and signal processing. This preliminary battle between old and new usually takes place at conferences, through the Internet and in the journals of the discipline. After a little more maturity has been acquired by the new concepts then archival publication as a scientific or engineering monograph may occur.

A new concept in control and signal processing is known to have arrived when sufficient material has evolved for the topic to be taught as a specialised tutorial workshop or as a course to undergraduate, graduates or industrial engineers. *Advanced Textbooks in Control and Signal Processing* are designed as a vehicle for the systematic presentation of course material for both popular and innovative topics in the discipline. It is hoped that prospective authors will welcome the opportunity to publish a structured and systematic presentation of some of the newer emerging control and signal processing technologies.

The essentials for any advanced course on control include a thorough understanding of state-space systems, both continuous- and discrete-time, the concepts of stochastic systems and insights into their optimal control systems. This textbook on *Discrete-time Stochastic Systems* by Torsten Söderström provides an invaluable introduction to these topics. It is a revised edition of an earlier Prentice Hall textbook which has benefited from a decade of classroom experience with Professor Söderström's course at Uppsala University, Sweden.

Apart from being used to support a full course, the text also has some interesting and useful features for the individual reader. The chapters are exceptionally well structured and can be used in a reference book fashion to instruct or review different technical topics, for example, spectral factorization. Unlike many linear stochastic control textbooks, Professor Söderström has given both time-domain and polynomial methods, proofs and techniques for topics like linear filtering and stochastic control systems. There are strong and fascinating links between these two approaches and it is invaluable to have them presented together in a single course textbook.

Every course lecturer likes to point their students to some topics which are a little more challenging and which might lead on to an interest deepening into research. Professor Söderström has included a chapter on nonlinear filtering which demonstrates how linear methods can be extended to deal with the more difficult nonlinear system problems. Each chapter is also accompanied by a bibliographical list of books and references to further reading for the interested reader.

The *Advanced Textbook in Control and Signal Processing* series seeks to create a set of books that are essential to a fundamental knowledge of the control and signal processing area. Professor Söderström's text is a welcome complement to other books in the series like Kamen and Su's *Introduction to Optimal Control* (ISBN 1-85233-133-X) and Williamson's *Discrete Time Signal Processing (*ISBN 1-85233-161-5) and we hope you will add *Discrete-time Stochastic Systems* to your library.

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Preface

This book has been written for graduate courses in stochastic dynamic systems. It has emerged from various lecture notes (in Swedish and in English) that I have compiled and used in different courses at Uppsala University since 1978.

The current text is a second edition of a book originally published by Prentice Hall International in 1994. All chapters have been revised. A number of typographical and other errors have been corrected. Various new material and results, including further problems, have been added.

The reader is assumed to be somewhat familiar with dynamic systems and stochastic processes. In particular, it is assumed that the reader has a working knowledge of the following areas (or is prepared to re-examine this background elsewhere, should it be necessary):

- Fundamentals of linear discrete-time systems (such as state space models and their relationships with transfer function operators and weighting functions).
- Fundamentals of probability theory (including Gaussian distributed random vectors and conditional probabilities).
- Fundamentals of linear algebra and matrix calculations.
- Fundamentals of stochastic processes (such as the concepts of covariance function and spectral density, particularly in discrete time).

In compiling the manuscript, I have taken inspiration from various sources, including other books. Some parts reflect my own findings and derivations. The bibliographical notes at the end of each chapter give hints for further reading. These notes have intentionally been kept brief and there is no ambition to supply comprehensive lists of references. The cited references do contain, in many cases, extensive publication lists. Many books deal with the fundamentals of linear stochastic systems, using analysis of state space models leading to the celebrated Kalman filter. Treatments using polynomial methods, as presented here, seem much less frequent in the literature. The same comment applies to extensions to nonlinear cases and higher-order statistics.

Most of the chapters contain problems to be treated as exercises by the reader. Many of these are of pen-and-pencil type, while others require numerical computation using computer programs. Some problems are straightforward illustrations of the results in the text. Several problems, however, are designed to present extensions and to give further insight. To achieve knowledge and understanding of the estimation and control of stochastic systems, it is of great importance that the user gains experience by his or her own work. I hope that the problem sections will stimulate the reader to gain such experience. When using the text for graduate courses I have let the students work with some of the problems as an integral part of the course examination.

The structure of the book is as follows.

Chapter 2 begins by giving a short review of probability theory. Some useful properties of conditional probabilities and densities, and of Gaussian variables are stated. One section is devoted to complex-valued Gaussian variables.

Various model descriptions of stochastic dynamic systems are illustrated in Chapter 3. Some basic definitions are reviewed briefly. The important concepts of a Markov process and a state vector in a stochastic setting are discussed. Properties of covariance functions, spectra, and higher-order moments are also presented.

Chapter 4 covers the analysis of linear stochastic systems, with an emphasis on second-order properties such as the propagation of the covariance function and the spectral density in a system. Spectral factorization, which is a very fundamental topic, is also treated. It is concerned with how to proceed from a specified spectrum to a filter description of a signal, so constructed that optimal prediction and control can be derived easily from that filter model.

The topic of Chapter 5 is optimal estimation, where "optimal" refers to mean square optimality (*i.e.* the estimation error variance is minimized). Under certain conditions more general performance measures are also minimized. The general theory is given, showing that the optimal estimate can often be described by a conditional expectation.

The celebrated Kalman filter is derived in Chapter 6, using the results of the previous chapter. Optimal prediction and smoothing algorithms (assuming that future data are available) are presented for a general linear state space model.

Optimal prediction for processes given in filter or polynomial form is presented in Chapter 7. The basic relations for Wiener filtering are also derived. A general (single input, single output) estimation problem is solved by applying the Wiener filter technique and using a polynomial formalism. Where a time-invariant input–output perspective is sufficient, this gives a convenient and interesting alternative to the state space methodology based on the Kalman filter. The solution is naturally the same in the time-invariant case using either approach.

Chapter 8 is devoted to an example in which a detailed treatment using both the state space and the polynomial approaches is examined. The optimal filters, error variances, frequency characteristics, and so on, are examined, and it is shown how they are influenced by the measurement noise, *etc.* The calculations illustrate the close connections between the polynomial and the state space approaches.

Many systems are inherently nonlinear. In Chapter 9, some nonlinear filters and nonlinear effects such as quantization are dealt with. The greater part of the chapter deals with the extended Kalman filter and some variants thereof.

The topic of Chapter 10 is the control of stochastic systems. It is first shown that the way in which uncertainties are introduced can make a distinct difference to the way in which the system should be optimally controlled. Next, the general optimal control problem is handled by using the principle of dynamic programming. As this approach, while theoretically very interesting, requires extreme amounts of computation, some suboptimal schemes are also discussed.

Finally, optimal control for linear systems that are perturbed by process and measurement noise is the topic of Chapter 11. This problem, known as linear quadratic Gaussian control, has an elegant solution based on the so-called separation theorem, and this is also described. Also, the use of polynomial formalism to derive some simple optimal controllers and some comparisons with state space formalism are included.

It is worth stressing here that the Glossary (pp. xiii–xv) contains items that appear frequently in the book.

Several people have contributed directly or indirectly to these lecture notes. As mentioned above, I have been teaching stochastic systems regularly in Uppsala and elsewhere for more than a decade. The feedback I have received from the many students over the years has been very valuable in compiling the manuscript. For the second edition I thank all those who have pointed out unclear points and errors in the first version. Special thanks go to Fredrik Sandquist, who detected a tricky mistake, to Dr Erik G. Larsson who pointed out a flaw in a proof, and to Professor Torbjörn Wigren who gave many suggestions for improving the chapter on nonlinear estimation and also provided some further exercises. Dr Egil Sviestins and Dr Niclas Bergman have contributed with valuable comments on nonlinear estimation.

Several students who recently used the text in a graduate course have pointed out various typos or unclear points. I am grateful to Emad Abd-ElRady, Richard Abrahamsson, Bharath Bhikkaji, Hong Cui, Mats Ekman, Kjartan Halvorsen, Bengt Johansson, Erik K. Larsson, Kaushik Mahata, Hans Norlander and Erik Ohlander for numerous valuable comments. Needless to say, the responsibility for any remaining errors rests upon me.

Last, but not least, I also acknowledge the nice and smooth cooperation with Springer-Verlag, and the persons who in some way or another have been involved in the production of this book: Professor Michael Grimble, Professor Michael Johnson, Ms. Catherine Drury, Mr Frank Holzwarth, Mr Oliver Jackson and Mr Peter Lewis.

To all the above people I express my sincere thanks.

In control, feedback is an essential concept. This is also true for writing books. I welcome comments by the readers, and can be reached by the email address ts@syscon.uu.se.

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