

# Contents

<b>Preface</b> .....	vii
<b>Contents</b> .....	ix
<b>1. What Do I Need to Know?</b> .....	1
1.1 Set Theory .....	2
1.2 Numbers .....	2
1.3 Sequences and Series .....	4
1.4 Functions and Continuity .....	7
1.5 Differentiation .....	10
1.6 Integration .....	12
1.7 Infinite Integrals .....	14
1.8 Calculus of Two Variables .....	17
<b>2. Complex Numbers</b> .....	19
2.1 Are Complex Numbers Necessary? .....	19
2.2 Basic Properties of Complex Numbers .....	21
<b>3. Prelude to Complex Analysis</b> .....	35
3.1 Why is Complex Analysis Possible? .....	35
3.2 Some Useful Terminology .....	37
3.3 Functions and Continuity .....	41
3.4 The $O$ and $o$ Notations .....	46
<b>4. Differentiation</b> .....	51
4.1 Differentiability .....	51
4.2 Power Series .....	61

4.3	Logarithms .....	71
4.4	Cuts and Branch Points .....	74
4.5	Singularities .....	75
<b>5.</b>	<b>Complex Integration</b> .....	<b>79</b>
5.1	The Heine–Borel Theorem .....	79
5.2	Parametric Representation .....	83
5.3	Integration .....	89
5.4	Estimation .....	99
5.5	Uniform Convergence .....	103
<b>6.</b>	<b>Cauchy’s Theorem</b> .....	<b>107</b>
6.1	Cauchy’s Theorem: A First Approach .....	107
6.2	Cauchy’s Theorem: A More General Version .....	111
6.3	Deformation .....	115
<b>7.</b>	<b>Some Consequences of Cauchy’s Theorem</b> .....	<b>119</b>
7.1	Cauchy’s Integral Formula .....	119
7.2	The Fundamental Theorem of Algebra .....	126
7.3	Logarithms .....	128
7.4	Taylor Series .....	131
<b>8.</b>	<b>Laurent Series and the Residue Theorem</b> .....	<b>137</b>
8.1	Laurent Series .....	137
8.2	Classification of Singularities .....	144
8.3	The Residue Theorem .....	146
<b>9.</b>	<b>Applications of Contour Integration</b> .....	<b>153</b>
9.1	Real Integrals: Semicircular Contours .....	153
9.2	Integrals Involving Circular Functions .....	158
9.3	Real Integrals: Jordan’s Lemma .....	161
9.4	Real Integrals: Some Special Contours .....	167
9.5	Infinite Series .....	176
<b>10.</b>	<b>Further Topics</b> .....	<b>183</b>
10.1	Integration of $f'/f$ ; Rouché’s Theorem .....	183
10.2	The Open Mapping Theorem .....	188
10.3	Winding Numbers .....	192
<b>11.</b>	<b>Conformal Mappings</b> .....	<b>195</b>
11.1	Preservation of Angles .....	195
11.2	Harmonic Functions .....	198
11.3	Möbius Transformations .....	203

---

11.4 Other Transformations .....	211
<b>12. Final Remarks</b> .....	<b>217</b>
12.1 Riemann's Zeta Function .....	217
12.2 Complex Iteration .....	221
<b>13. Solutions to Exercises</b> .....	<b>225</b>
<b>Bibliography</b> .....	<b>255</b>
<b>Index</b> .....	<b>257</b>