Preface

The idea for this monograph was born out of the desire to collate the results from two distinct strands of the authors' research with the common theme of the application of the method of matched asymptotic expansions to problems arising in reaction-diffusion theory.

In Part I, the method of matched asymptotic expansions (MAE) is used to obtain the complete structure of the solution to reaction-diffusion equations of the Fisher-Kolmogorov type for large-t (dimensionless time), which exhibit the formation of a permanent form travelling wave (PTW) structure. In particular, the wave speed for the large-t PTW, the correction to the wave speed and the rate of convergence of the solution onto the PTW are obtained. The primary focus of Chapters 2-4 is the scalar Fisher-Kolmogorov equation with either the generalized Fisher nonlinearity or the mth-order ($m > 1$) Fisher nonlinearity while in Chapter 5 the analysis is extended by consideration of a system of Fisher-Kolmogorov equations. The methodology developed is flexible and has wide applicability to scalar and systems of Fisher-Kolmogorov equations in one or higher spatial dimensions. The method of matched asymptotic expansions has also been used successfully to give information about the structure and propagation speed of accelerating phase wave (PHW) structures which can evolve in reaction-diffusion equations (see Needham and Barnes [56]) and nonlinear diffusion equations of Fisher-Kolmogorov type. The approach presented in this part of the monograph is based on the results obtained in the series of papers by Leach and Needham [32],[33],[34], Leach, Needham and Kay [35],[37],[36] and Smith, Needham and Leach [65].

In Part II we analyze a class of singular (in the sense that the nonlinearities are not Lipschitz continuous) reaction-diffusion equations. These reaction-diffusion equations can display a wide range of behaviour including:

(i) Solutions which decay to zero with contracting support in finite t (say t_c).
(ii) Spatially uniform solutions which grow algebraically in t.
(iii) Permanent form travelling waves which are excitable (rather than of Fisher-Kolmogorov type).

A detailed analysis is presented of the permanent form travelling wave theory in Chapter 7 and of the initial-boundary value problem in Chapters 8 and 9. This analysis requires substantial modifications to the standard theory developed for regular reaction-diffusion equations. In particular, we develop, via the method of MAE, the asymptotic structure as $t \to 0$ and as $t \to \infty$ (or $t \to t_c^-$) over all parameter values. A system of singular reaction-diffusion equations is considered in Chapter 10, with particular emphasis on the asymptotic development of the solution as $t \to 0$. The approach presented in this part of the monograph is based on the results obtained in the series of papers by McCabe, Leach and Needham [38], [39],[40],[41] and [42].

This monograph contains a wealth of results and methodologies which are applicable to a wide range of related problems arising in reaction-diffusion theory. In particular, the regions (with the analysis of their associated boundary-value problems) that constitute the asymptotic structures presented can be considered as the building blocks of the asymptotic structures of other related problems. Hence this monograph can be viewed both as a handbook and as a detailed description of methodology.

Throughout we use the nomenclature of the theory of matched asymptotic expansions, as given in Van Dyke [70]. The monograph assumes a general knowledge of perturbation methods (see for example Lagerstrom and Casten [30], Nayfeh [53], Georgescu [18] and Hinch [26]), dynamical systems theory (see for example Perko [59] and Wiggins [72]) and reaction-diffusion theory (see for example Britton [10], Fife [14] and Volpert *et al* [71]).

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Part I The Evolution of Travelling Waves in Scalar Fisher-Kolmogorov Equations