Advanced Textbooks in Control and Signal Processing

Series Editors

Professor Michael J. Grimble, Professor of Industrial Systems and Director
Professor Michael A. Johnson, Professor Emeritus of Control Systems and Deputy Director
Industrial Control Centre, Department of Electronic and Electrical Engineering,
University of Strathclyde, Graham Hills Building, 50 George Street, Glasgow G1 1QE, UK

Other titles published in this series:

Genetic Algorithms
K.F. Man, K.S. Tang and S. Kwong

Neural Networks for Modelling and Control of Dynamic Systems
M. Nørgaard, O. Ravn, L.K. Hansen and N.K. Poulsen

Modelling and Control of Robot Manipulators (2nd Edition)
L. Sciavicco and B. Siciliano

Fault Detection and Diagnosis in Industrial Systems
L.H. Chiang, E.L. Russell and R.D. Braatz

Soft Computing
L. Fortuna, G. Rizzotto, M. Lavorgna, G. Nunnari, M.G. Xibilia and R. Caponetto

Statistical Signal Processing
T. Chonavel

Discrete-time Stochastic Processes (2nd Edition)
T. Söderström

Parallel Computing for Real-time Signal Processing and Control
M.O. Tokhi, M.A. Hossain and M.H. Shaheed

Multivariable Control Systems
P. Albertos and A. Sala

Control Systems with Input and Output Constraints
A.H. Glattfelder and W. Schaufelberger

Analysis and Control of Non-linear Process Systems
K. Hangos, J. Bokor and G. Szederkényi

Model Predictive Control (2nd Edition)
E.F. Camacho and C. Bordons

Principles of Adaptive Filters and Self-learning Systems
A. Zaknich

Digital Self-tuning Controllers
V. Bobál, J. Böhm, J. Fessl and J. Macháček

Control of Robot Manipulators in Joint Space
R. Kelly, V. Santibáñez and A. Loría

Active Noise and Vibration Control
M.O. Tokhi
Publication due November 2005

D.-W. Gu, P. Hr. Petkov and M. M. Konstantinov

Robust Control Design with MATLAB®

With 288 Figures

Springer

Da-Wei Gu, PhD, DIC, CEng
Engineering Department, University of Leicester, University Road, Leicester, LE1 7RH, UK

Petko Hristov Petkov, PhD
Department of Automatics, Technical University of Sofia, 1756 Sofia, Bulgaria

Mihail Mihaylov Konstantinov, PhD
University of Architecture, Civil Engineering and Geodesy,
1 Hristo Smirnenski Blvd., 1046 Sofia, Bulgaria

British Library Cataloguing in Publication Data
Gu, D.-W.
Robust control design with MATLAB. - (Advanced textbooks in control and signal processing)
1. MATLAB (Computer file) 2. Robust control 3. Control theory
I. Title II. Petkov, P. Hr (Petko Hr.), 1948-
III. Konstantinov, M. M. (Mihail M.), 1948-
629.8'312
ISBN-10: 1852339837

Library of Congress Control Number: 2005925110

Apart from any fair dealing for the purposes of research or private study, or criticism or review, as permitted under the Copyright, Designs and Patents Act 1988, this publication may only be reproduced, stored or transmitted, in any form or by any means, with the prior permission in writing of the publishers, or in the case of reprographic reproduction in accordance with the terms of licences issued by the Copyright Licensing Agency. Enquiries concerning reproduction outside those terms should be sent to the publishers.

Advanced Textbooks in Control and Signal Processing series ISSN 1439-2232
ISBN-10: 1-85233-983-7
ISBN-13: 978-1-85233-983-8
Springer Science+Business Media
springeronline.com

© Springer-Verlag London Limited 2005

MATLAB® and Simulink® are the registered trademarks of The MathWorks, Inc., 3 Apple Hill Drive, Natick, MA 01760-2098, U.S.A. http://www.mathworks.com

The software disk accompanying this book and all material contained on it is supplied without any warranty of any kind. The publisher accepts no liability for personal injury incurred through use or misuse of the disk.

The use of registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant laws and regulations and therefore free for general use.

The publisher makes no representation, express or implied, with regard to the accuracy of the information contained in this book and cannot accept any legal responsibility or liability for any errors or omissions that may be made.

Typesetting: Camera ready by authors
Production: LE-TEX Jelonek, Schmidt & Vöckler GbR, Leipzig, Germany
Printed in Germany
69/3141-543210 Printed on acid-free paper SPIN 11309833

To our families

Series Editors' Foreword

The topics of control engineering and signal processing continue to flourish and develop. In common with general scientific investigation, new ideas, concepts and interpretations emerge quite spontaneously and these are then discussed, used, discarded or subsumed into the prevailing subject paradigm. Sometimes these innovative concepts coalesce into a new sub-discipline within the broad subject tapestry of control and signal processing. This preliminary battle between old and new usually takes place at conferences, through the Internet and in the journals of the discipline. After a little more maturity has been acquired by the new concepts then archival publication as a scientific or engineering monograph may occur.

A new concept in control and signal processing is known to have arrived when sufficient material has evolved for the topic to be taught as a specialised tutorial workshop or as a course to undergraduate, graduate or industrial engineers. *Advanced Textbooks in Control and Signal Processing* are designed as a vehicle for the systematic presentation of course material for both popular and innovative topics in the discipline. It is hoped that prospective authors will welcome the opportunity to publish a structured and systematic presentation of some of the newer emerging control and signal processing technologies in the textbook series.

It is always interesting to look back at how a particular field of control systems theory developed. The impetus for change and realization that a new era in a subject is dawning always seems to be associated with short, sharp papers that make the academic community think again about the prevalent theoretical paradigm. In the case of the evolution of robust control theory, the conference papers of Zames (circa. 1980) on robustness and the very short paper of Doyle on the robustness of linear quadratic Gaussian control systems seem to stand as landmarks intimating that control theory was going to change direction again. And the change did come; all through the 1980s came a steady stream of papers re-writing control theory, introducing system uncertainty, H_∞ robust control and μ-synthesis as part of a new control paradigm.

Change, however did not come easily to the industrial applications community because the new theories and methods were highly mathematical. In the early stages even the classical feedback diagram which so often opened control engineering courses was replaced by a less intuitively obvious diagram. Also it

was difficult to see the benefits to be gained from the new development. Throughout the 1990s the robust control theory and methods consolidated and the first major textbooks and software toolboxes began to appear. Experience with some widely disseminated benchmark problems such as control design for distillation columns, the control design for hard-disk drives, and the inverted-pendulum control problem helped the industrial community see how to apply the new method and the control benefits that accrued.

This advanced course textbook on robust control system design using MATLAB® by Da-Wei Gu, Petko Petkov and Mihail Konstantinov has arrived at a very opportune time. More than twenty years of academic activity in the robust control field forms the bedrock on which this course book and its set of insightful applications examples are developed. Part I of the volume presents the theory – a systematic presentation of: systems notation, uncertainty modelling, robust design specification, the H_∞ design method, H_∞ loop shaping, μ-analysis and synthesis and finally the algorithms for providing the low-order controllers that will be implemented. This is a valuable and concise presentation of all the necessary theoretical concepts prior to their application which is covered in Part II.

Inspired by the adage "practice makes perfect", Part II of the volume comprises six fully worked-out extended examples. To learn how to apply the complex method of H_∞ design and μ-synthesis there can be no surer route than to work through a set of carefully scripted examples. In this volume, the examples range from the academic mass-damper-spring system through to the industrially relevant control of a distillation column and a flexible manipulator system. The benchmark example of the ubiquitous hard-disk drive control system is also among the examples described. The MATLAB® tools of the Robust Control Toolbox, the Control System Toolbox and Simulink® are used in these application examples. The CD-ROM contains all the necessary files and instructions together with a pdf containing colour reproductions of many of the figures in the book.

In summary, after academic development of twenty years or so, the robust control paradigm is now fully fledged and forms a vital component of advanced control engineering courses. This new volume in our series of advanced control and signal processing course textbooks on applying the methods of H_∞ and μ-synthesis control design will be welcomed by postgraduate students, lecturers and industrial control engineers alike.

M.J. Grimble and M.A. Johnson
Glasgow, Scotland, U.K.
February 2005

Preface

Robustness has been an important issue in control-systems design ever since 1769 when James Watt developed his flyball governor. A successfully designed control system should be always able to maintain stability and performance level in spite of uncertainties in system dynamics and/or in the working environment to a certain degree. Design requirements such as gain margin and phase margin in using classical frequency-domain techniques are solely for the purpose of robustness. The robustness issue was not that prominently considered during the period of 1960s and 1970s when system models could be much more accurately described and design methods were mainly mathematical optimisations in the time domain. Due to its importance, however, the research on robust design has been going on all the time. A breakthrough came in the late 1970s and early 1980s with the pioneering work by Zames [170] and Zames and Francis [171] on the theory, now known as the $\mathcal{H}_\infty$ optimal control theory. The $\mathcal{H}_\infty$ optimisation approach and the μ-synthesis/analysis method are well developed and elegant. They provide systematic design procedures of robust controllers for linear systems, though the extension into nonlinear cases is being actively researched.

Many books have since been published on $\mathcal{H}_\infty$ and related theories and methods [26, 38, 65, 137, 142, 145, 174, 175]. The algorithms to implement the design methods are readily available in software packages such as MATLAB® and Slicot [119]. However, from our experience in teaching and research projects, we have felt that a reasonable percentage of people, students as well as practising engineers, still have difficulties in applying the $\mathcal{H}_\infty$ and related theory and in using MATLAB® routines. The mathematics behind the theory is quite involved. It is not straightforward to formulate a practical design problem, which is usually nonlinear, into the $\mathcal{H}_\infty$ or μ design framework and then apply MATLAB® routines. This hinders the application of such a powerful theory. It also motivated us to prepare this book.

This book is for people who want to learn how to deal with robust control-system design problems but may not want to research the relevant theoretic developments. Methods and solution formulae are introduced in the first part

of the book, but kept to a minimum. The majority of the book is devoted to several practical design case studies (Part II). These design examples, ranging from teaching laboratory experiments such as a mass-damper-spring system to complex systems such as a supersonic rocket autopilot and a flexible-link manipulator, are discussed with detailed presentations. The design exercises are all conducted using the new *Robust Control Toolbox v3.0* and are in a hands-on, tutorial manner. Studying these examples with the attached MATLAB® and Simulink® programs (170 plus M- and MDL-files) used in all designs will help the readers learn how to deal with nonlinearities involved in the system, how to parameterise dynamic uncertainties and how to use MATLAB® routines in the analysis and design, *etc.*. It is also hoped that by going through these exercises the readers will understand the essence of robust control system design and develop their own skills to design real, industrial, robust control systems.

The readership of this book is postgraduates and control engineers, though senior undergraduates may use it for their final-year projects. The material included in the book has been adopted in recent years for MSc and PhD engineering students at Leicester University and at the Technical University of Sofia. The design examples are independent of each other. They have been used extensively in the laboratory projects on the course *Robust and Optimal Control Systems* taught in a masters programme in the Technical University of Sofia.

The authors are indebted to several people and institutions who helped them in the preparation of the book. We are particularly grateful to The MathWorks, Inc. for their continuous support, to Professor Sigurd Skogestad of Norwegian University of Science and Technology who kindly provided the nonlinear model of the Distillation Column and to Associate Professor Georgi Lehov from Technical University of Russe, Bulgaria, who developed the uncertainty model of the Flexible-Link Manipulator.

Using the CD ROM

The attached CD ROM contains six folders with M- and MDL-files intended for design, analysis and simulation of the six design examples, plus a pdf file with colour hypertext version of the book. In order to use the M- and MDL-files the reader should have at his (her) disposition of MATLAB® v7.0.2 with Robust Control Toolbox v 3.0, Control System Toolbox v6.1 and Simulink® v6.1. Further information on the use of the files can be found in the file Readme.m on the disc.

Contents

Part I Basic Methods and Theory

1 Introduction . . . 3
- 1.1 Control-system Representations . . . 4
- 1.2 System Stabilities . . . 6
- 1.3 Coprime Factorisation and Stabilising Controllers . . . 7
- 1.4 Signals and System Norms . . . 9
  - 1.4.1 Vector Norms and Signal Norms . . . 9
  - 1.4.2 System Norms . . . 10

2 Modelling of Uncertain Systems . . . 13
- 2.1 Unstructured Uncertainties . . . 13
- 2.2 Parametric Uncertainty . . . 17
- 2.3 Linear Fractional Transformations . . . 20
- 2.4 Structured Uncertainties . . . 23

3 Robust Design Specifications . . . 25
- 3.1 Small-gain Theorem and Robust Stabilisation . . . 25
- 3.2 Performance Consideration . . . 28
- 3.3 Structured Singular Values . . . 29

4 $\mathcal{H}_\infty$ Design . . . 35
- 4.1 Mixed Sensitivity $\mathcal{H}_\infty$ Optimisation . . . 35
- 4.2 2-Degree-Of-Freedom $\mathcal{H}_\infty$ Design . . . 38
- 4.3 $\mathcal{H}_\infty$ Suboptimal Solutions . . . 39
  - 4.3.1 Solution Formulae for Normalised Systems . . . 39
  - 4.3.2 Solution to S-over-KS Design . . . 43
  - 4.3.3 The Case of $D_{22} \neq 0$ . . . 44
  - 4.3.4 Normalisation Transformations . . . 45
  - 4.3.5 Direct Formulae for $\mathcal{H}_\infty$ Suboptimal Central Controller 47
- 4.4 Formulae for Discrete-time Cases . . . 50

5 $\mathcal{H}_\infty$ Loop-shaping Design Procedures 55
5.1 Robust Stabilisation Against Normalised Coprime Factor Perturbations 56
5.2 Loop-shaping Design Procedures 58
5.3 Formulae for the Discrete-time Case 61
5.3.1 Normalised Coprime Factorisation of Discrete-time Plant 61
5.3.2 Robust Controller Formulae 62
5.3.3 The Strictly Proper Case 63
5.3.4 On the Three DARE Solutions 65
5.4 A Mixed Optimisation Design Method with LSDP 67

6 μ-Analysis and Synthesis 71
6.1 Consideration of Robust Performance 71
6.2 μ-Synthesis: D-K Iteration Method 74
6.3 μ-Synthesis: μ-K Iteration Method 77

7 Lower-order Controllers 79
7.1 Absolute-error Approximation Methods 80
7.1.1 Balanced Truncation Method 81
7.1.2 Singular Perturbation Approximation 82
7.1.3 Hankel-norm Approximation 83
7.1.4 *Remarks* 85
7.2 Reduction via Fractional Factors 86
7.3 Relative-error Approximation Methods 90
7.4 Frequency-weighted Approximation Methods 92

Part II Design Examples

8 Robust Control of a Mass-Damper-Spring System 101
8.1 System Model 101
8.2 Frequency Analysis of Uncertain System 107
8.3 Design Requirements of Closed-loop System 108
8.4 System Interconnections 112
8.5 Suboptimal $\mathcal{H}_\infty$ Controller Design 115
8.6 Analysis of Closed-loop System with K_{hin} 117
8.7 $\mathcal{H}_\infty$ Loop-shaping Design 125
8.8 Assessment of $\mathcal{H}_\infty$ Loop-shaping Design 128
8.9 μ-Synthesis and D-K Iterations 131
8.10 Robust Stability and Performance of K_{mu} 141
8.11 Comparison of $\mathcal{H}_\infty$, $\mathcal{H}_\infty$ LSDP and μ-controllers 150
8.12 Order Reduction of μ-controller 158
8.13 Conclusions 162

9 A Triple Inverted Pendulum Control-system Design 163
9.1 System Description 164
9.2 Modelling of Uncertainties 167
9.3 Design Specifications 180
9.4 System Interconnections 182
9.5 $\mathcal{H}_\infty$ Design 186
9.6 μ-Synthesis 191
9.7 Nonlinear System Simulation 199
9.8 Conclusions 202

10 Robust Control of a Hard Disk Drive 203
10.1 Hard Disk Drive Servo System 203
10.2 Derivation of Uncertainty Model 209
10.3 Closed-loop System-design Specifications 215
10.4 System Interconnections 218
10.5 Controller Design in Continuous-time 219
10.5.1 μ-Design 221
10.5.2 $\mathcal{H}_\infty$ Design 228
10.5.3 $\mathcal{H}_\infty$ Loop-shaping Design 228
10.6 Comparison of Designed Controllers 229
10.7 Controller-order Reduction 237
10.8 Design of Discrete-time Controller 239
10.9 Nonlinear System Simulation 244
10.10Conclusions 247

11 Robust Control of a Distillation Column 249
11.1 Introduction 249
11.2 Dynamic Model of the Distillation Column 250
11.3 Uncertainty Modelling 254
11.4 Closed-loop System-performance Specifications 256
11.5 Open-loop and Closed-loop System Interconnections 261
11.6 Controller Design 261
11.6.1 Loop-shaping Design 262
11.6.2 μ-Synthesis 271
11.7 Nonlinear System Simulation 283
11.8 Conclusions 286

12 Robust Control of a Rocket 289
12.1 Rocket Dynamics 289
12.2 Uncertainty Modelling 301
12.3 Performance Requirements 306
12.4 $\mathcal{H}_\infty$ Design 310
12.5 μ-Synthesis 315
12.6 Discrete-time μ-Synthesis 324
12.7 Simulation of the Nonlinear System 328

12.8 Conclusions 332

13 Robust Control of a Flexible-Link Manipulator 335
13.1 Dynamic Model of the Flexible Manipulator 336
13.2 A Linear Model of the Uncertain System 339
13.3 System-performance Specifications 355
13.4 System Interconnections 359
13.5 Controller Design and Analysis 361
13.6 Nonlinear System Simulations 372
13.7 Conclusions 375

References 377

Index 387