

---

# Contents

<b>Preface</b> .....	VII
<b>Notation and Definitions</b> .....	XV

---

<b>Part I Multiparametric Programming</b>	
---	--

---

<b>Multiparametric Programming: a Geometric Approach</b> .....	3
1.1 Introduction .....	4
1.2 General Results for Multiparametric Nonlinear Programs .....	5
1.3 Multiparametric Linear Programming .....	11
1.3.1 Geometric Algorithm for mp-LP .....	13
1.3.2 Determining the Affine Subspace $\mathcal{K}$ .....	13
1.3.3 Determining the Critical Regions .....	16
1.3.4 Degeneracy .....	20
1.3.5 A Summary of the mp-LP Algorithm .....	25
1.3.6 Complexity Analysis .....	27
1.4 Multiparametric Quadratic Programming .....	31
1.4.1 Geometric Algorithm for mp-QP .....	32
1.4.2 A Summary of the mp-QP Algorithm .....	35
1.4.3 Propagation of the Set of Active Constraints .....	36
1.4.4 Continuity, Convexity and $C^{(1)}$ Properties .....	36
1.4.5 Complexity Analysis .....	40
1.4.6 Other Algorithms for solving mp-QP .....	41
1.5 Multiparametric Mixed-Integer Linear Programming .....	43
1.5.1 Geometric Algorithm for mp-MILP .....	43
1.5.2 Theoretical Results .....	46
1.6 Multiparametric Mixed-Integer Quadratic Programming .....	47

---

**Part II Optimal Control of Linear Systems**

---

**Constrained Finite Time Optimal Control** . . . . . 51

    2.1 Problem Formulation . . . . . 52

    2.2 State Feedback Solution of CFTOC, 2-Norm Case . . . . . 54

        2.2.1 Complexity Analysis . . . . . 57

        2.2.2 Examples . . . . . 58

    2.3 State Feedback Solution of CFTOC,  $1, \infty$ -Norm Case . . . . . 62

        2.3.1 Complexity Analysis . . . . . 65

        2.3.2 Example . . . . . 65

    2.4 Time Varying Systems . . . . . 66

**Constrained Infinite Time Optimal Control** . . . . . 71

    3.1 Solution to the Infinite Time Constrained LQR Problem. . . . . 72

    3.2 Examples. . . . . 73

**Receding Horizon Control** . . . . . 75

    4.1 Introduction . . . . . 76

    4.2 Problem Formulation . . . . . 77

        4.2.1 Stability of RHC . . . . . 79

        4.2.2 Stability,  $\infty$ -Norm case. . . . . 81

    4.3 State Feedback Solution of RHC, 2-Norm Case . . . . . 83

        4.3.1 Examples . . . . . 84

    4.4 State Feedback Solution of RHC,  $1, \infty$ -Norm Case . . . . . 90

        4.4.1 Idle Control and Multiple Optima . . . . . 90

        4.4.2 Examples . . . . . 91

    4.5 On-Line Computation Time . . . . . 98

    4.6 RHC Extensions . . . . . 98

**Constrained Robust Optimal Control** . . . . . 103

    5.1 Introduction . . . . . 104

    5.2 Problem Formulation . . . . . 105

    5.3 State Feedback Solution to CROC Problems . . . . . 107

        5.3.1 Preliminaries on Multiparametric Programming . . . . . 108

        5.3.2 Closed Loop CROC . . . . . 110

        5.3.3 Open Loop CROC . . . . . 111

        5.3.4 Solution to CL-CROC and OL-CROC via mp-MILP . . 112

    5.4 Robust Receding Horizon Control . . . . . 113

    5.5 Examples. . . . . 113

**Reducing On-line Complexity** . . . . . 117

    6.1 Introduction . . . . . 118

    6.2 Efficient On-Line Algorithms. . . . . 118

        6.2.1 Efficient Implementation,  $1, \infty$ -Norm Case . . . . . 120

        6.2.2 Efficient Implementation, 2-Norm Case . . . . . 121

6.3	Example	128
6.3.1	CFTOC based on LP	128
6.3.2	CFTOC based on QP	129

---

## Part III Optimal Control of Hybrid Systems

---

<b>Hybrid Systems</b>	133	
7.1	Introduction	134
7.2	Mixed Logic Dynamical (MLD) Systems	136
7.3	HYSDEL	139
7.4	Theoretical Properties of PWA Systems	139
<b>Constrained Optimal Control for Hybrid Systems</b>	143	
8.1	Introduction	144
8.2	Problem Formulation	145
8.3	State Feedback Solution of CFTOC, 2-Norm Case	146
8.4	State Feedback Solution of CFTOC, $1, \infty$ -Norm Case	152
8.5	Efficient Computation of the Optimal Control Input	153
8.6	Efficient Computation of the State Feedback Solution	154
8.7	Computation of the State Feedback Solution, $1, \infty$ -Norm Case	156
8.7.1	Example	156
8.8	Computation of the State Feedback Solution, 2-Norm Case	157
8.8.1	Preliminaries and Basic Steps	158
8.8.2	Efficient Dynamic Program for the Computation of the Solution	163
8.8.3	Example	164
8.9	Receding Horizon Control	165
8.9.1	Convergence	166
8.9.2	Extensions	167
8.9.3	Examples	167

---

## Part IV Applications

---

<b>Ball and Plate</b>	177	
9.1	Ball and Plate Dynamic Model	178
9.2	Constrained Optimal Control	180
9.2.1	Tuning	181
9.3	Experimental Setup	181

**Traction Control** . . . . . 185

- 10.1 Introduction . . . . . 186
- 10.2 Vehicle Model . . . . . 187
  - 10.2.1 Discrete-Time Hybrid Model of the Vehicle . . . . . 188
- 10.3 Constrained Optimal Control . . . . . 190
- 10.4 Controller Design . . . . . 192
  - 10.4.1 Tuning . . . . . 193
  - 10.4.2 Combustion Torque Delay . . . . . 193
- 10.5 Motivation for Hybrid Control . . . . . 194
- 10.6 Experimental Setup and Results . . . . . 194
- 10.7 HYSDEL Hybrid Model . . . . . 195

---

**Bibliography**

---

**References** . . . . . 197