

---

# Contents

<b>Introduction</b> .....	1
<b>I Preliminaries</b> .....	3
1 Basic Definitions and Notation .....	3
2 Martingales .....	7
3 The Poisson Process and Brownian Motion .....	12
4 Lévy Processes .....	19
5 Why the Usual Hypotheses? .....	34
6 Local Martingales .....	37
7 Stieltjes Integration and Change of Variables .....	39
8 Naïve Stochastic Integration Is Impossible .....	43
Bibliographic Notes .....	44
Exercises for Chapter I .....	45
<b>II Semimartingales and Stochastic Integrals</b> .....	51
1 Introduction to Semimartingales .....	51
2 Stability Properties of Semimartingales .....	52
3 Elementary Examples of Semimartingales .....	54
4 Stochastic Integrals .....	56
5 Properties of Stochastic Integrals .....	60
6 The Quadratic Variation of a Semimartingale .....	66
7 Itô's Formula (Change of Variables) .....	78
8 Applications of Itô's Formula .....	84
Bibliographic Notes .....	92
Exercises for Chapter II .....	94
<b>III Semimartingales and Decomposable Processes</b> .....	101
1 Introduction .....	101
2 The Classification of Stopping Times .....	103
3 The Doob-Meyer Decompositions .....	105
4 Quasimartingales .....	116

XII Contents

5	Compensators . . . . .	118
6	The Fundamental Theorem of Local Martingales . . . . .	124
7	Classical Semimartingales . . . . .	127
8	Girsanov's Theorem . . . . .	131
9	The Bichteler-Dellacherie Theorem . . . . .	143
	Bibliographic Notes . . . . .	147
	Exercises for Chapter III . . . . .	147
<b>IV</b>	<b>General Stochastic Integration and Local Times . . . . .</b>	<b>153</b>
1	Introduction . . . . .	153
2	Stochastic Integration for Predictable Integrand . . . . .	153
3	Martingale Representation . . . . .	178
4	Martingale Duality and the Jacod-Yor Theorem on Martingale Representation . . . . .	193
5	Examples of Martingale Representation . . . . .	200
6	Stochastic Integration Depending on a Parameter . . . . .	205
7	Local Times . . . . .	210
8	Azéma's Martingale . . . . .	227
9	Sigma Martingales . . . . .	233
	Bibliographic Notes . . . . .	235
	Exercises for Chapter IV . . . . .	236
<b>V</b>	<b>Stochastic Differential Equations . . . . .</b>	<b>243</b>
1	Introduction . . . . .	243
2	The $\underline{H}^p$ Norms for Semimartingales . . . . .	244
3	Existence and Uniqueness of Solutions . . . . .	249
4	Stability of Stochastic Differential Equations . . . . .	257
5	Fisk-Stratonovich Integrals and Differential Equations . . . . .	270
6	The Markov Nature of Solutions . . . . .	291
7	Flows of Stochastic Differential Equations: Continuity and Differentiability . . . . .	301
8	Flows as Diffeomorphisms: The Continuous Case . . . . .	310
9	General Stochastic Exponentials and Linear Equations . . . . .	321
10	Flows as Diffeomorphisms: The General Case . . . . .	328
11	Eclectic Useful Results on Stochastic Differential Equations . . . . .	338
	Bibliographic Notes . . . . .	347
	Exercises for Chapter V . . . . .	349
<b>VI</b>	<b>Expansion of Filtrations . . . . .</b>	<b>355</b>
1	Introduction . . . . .	355
2	Initial Expansions . . . . .	356
3	Progressive Expansions . . . . .	369
4	Time Reversal . . . . .	377
	Bibliographic Notes . . . . .	383
	Exercises for Chapter VI . . . . .	384

<b>References</b> .....	389
<b>Symbol Index</b> .....	403
<b>Subject Index</b> .....	407