

Contents

Introduction	3
1 Development of the theory of motion for systems with Coulomb friction	11
1.1 Coulomb's law of friction	11
1.2 Main peculiarities of systems with Coulomb friction and the specific problems of the theory of motion	12
1.2.1 The principle peculiarity	13
1.2.2 Non-closed system of equations for the dynamics of systems with friction and the problem of deriving these equations	14
1.2.3 Non-correctness of the equations for systems with friction and the problem of solving Painlevé's paradoxes	15
1.2.4 The problem of determining the forces of friction acting on particles	17
1.2.5 Retaining the state of rest and transition to motion	18
1.2.6 The problem of determining the property of self-braking	19
1.2.7 Appearance of self-excited oscillations	19
1.3 Various interpretations of Painlevé's paradoxes	20
1.4 Principles of the general theory of systems with Coulomb friction	25
1.5 Laws of Coulomb friction and the theory of frictional self-excited oscillations	32

2	Systems with a single degree of freedom and a single frictional pair	37
2.1	Lagrange's equations with a removed contact constraint . . .	37
2.2	Kinematic expression for slip with rolling	44
2.2.1	Velocity of slip and the velocities of change of the contact place due to the trace of the contact	44
2.2.2	Angular velocity	45
2.3	Equation for the constraint force and Painlevé's paradoxes .	49
2.3.1	Solution for the acceleration and the constraint force	50
2.3.2	Criterion for the paradoxes	51
2.4	Immovable contact and transition to slipping	53
2.5	Self-braking and the angle of stagnation	57
2.5.1	The case of no paradoxes	58
2.5.2	The case of paradoxes ($\mu L > 1$)	64
3	Accounting for dry friction in mechanisms. Examples of single-degree-of-freedom systems with a single frictional pair	67
3.1	Two simple examples	67
3.1.1	First example	67
3.1.2	Second example	69
3.2	The Painlevé-Klein extended scheme	70
3.2.1	Differential equations of motion, expression for the reaction force, condition for the paradoxes and the law of motion	72
3.2.2	Immovable contact and transition to slip	74
3.2.3	The stagnation angle and the property of self-braking in the case of no paradoxes	75
3.2.4	Self-braking under the condition of paradoxes	77
3.3	Stacker	79
3.3.1	Pure rolling of the rigid body model	79
3.3.2	Slip of the driving wheel for the rigid body model .	82
3.3.3	Speed-up of stacker	84
3.3.4	Pure rolling in the case of tangential compliance . .	85
3.3.5	Rolling with account of compliance	87
3.3.6	Speed-up with account of compliance	88
3.3.7	Numerical example	91
3.4	Epicyclic mechanism with cylindric teeth of the involute gearing	94
3.4.1	Differential equation of motion, equations for the reaction force and the conditions for paradoxes.	95
3.4.2	Relationships between the torques at rest and in the transition to motion	100
3.4.3	Regime of uniform motion	103
3.5	Gear transmission with immovable rotation axes	103

3.5.1	Differential equations of motion and the condition for absence of paradoxes	104
3.5.2	Regime of uniform motion	106
3.5.3	Transition from the state of rest to motion	109
3.6	Crank mechanism	110
3.6.1	Equation of motion and reaction force	110
3.6.2	Condition for complete absence of paradoxes	112
3.6.3	The property of self-braking in the case of no paradoxes	114
3.7	Link mechanism of a planing machine	115
3.7.1	Differential equations of motion and the expression for the reaction force	115
3.7.2	Feasibility of Painlevé's paradoxes	119
3.7.3	The property of self-braking	121
3.7.4	Numerical example	123
4	Systems with many degrees of freedom and a single frictional pair. Solving Painlevé's paradoxes	125
4.1	Lagrange's equations with a removed constraint	125
4.2	Equation for the constraint force, differential equation of motion and the criterion of paradoxes	128
4.2.1	Determination of the constraint force and acceleration	128
4.2.2	Criterion of Painlevé's paradoxes	131
4.3	Determination of the true motion	132
4.3.1	Limiting process	133
4.3.2	True motions under the paradoxes	137
4.4	True motions in the Painlevé-Klein problem in paradoxical situations	141
4.4.1	Equations for the reaction force	142
4.4.2	True motions for the paradoxes	143
4.5	Elliptic pendulum	145
4.6	The Zhukovsky-Froude pendulum	148
4.6.1	Equation for the reaction force and condition for the non-existence of the solution	150
4.6.2	The equilibrium position and free oscillations	152
4.6.3	Regime of joint rotation of the journal and the pin	153
4.7	A condition of instability for the stationary regime of metal cutting	155
4.7.1	Derivation of the equations of motion	155
4.7.2	Solving the equations	157
4.7.3	Relationship between instability of cutting and Painlevé's paradox	159
4.7.4	Boring with an axial feed	161

5	Systems with several frictional pairs. Painlevé's law of friction. Equations for the perturbed motion taking account of contact compliance	163
5.1	Equations for systems with Coulomb friction	163
5.1.1	System with removed constraints	163
5.1.2	Solving the main system	166
5.1.3	The case of $n = 1, m = 1$	169
5.2	Mathematical description of the Painlevé law of friction	170
5.2.1	Accelerations due to two systems of external forces	170
5.2.2	Improved Painlevé's equations	172
5.2.3	Improved Painlevé's theorem	174
5.3	Forces of friction in the Painlevé-Klein problem	176
5.4	The contact compliance and equations of perturbed trajectories	177
5.4.1	Lagrange's equations for systems with elastic contact joints	177
5.4.2	Equations for perturbed reaction forces	179
5.5	Painlevé's scheme with two frictional pairs	181
5.5.1	Lagrange's equations, reaction forces and the equations of motion with eliminated reaction forces	182
5.5.2	Feasibility of Painlevé's paradoxes	184
5.5.3	Expressions for the frictional force in terms of the friction coefficients	185
5.5.4	Painlevé's scheme for compliant contacts	186
5.6	Sliders of metal-cutting machine tools	187
5.6.1	Derivation of equations of motion and expressions for the reaction forces	187
5.6.2	Signs of the reaction forces and feasibility of paradoxes	189
5.6.3	Forces of friction	191
5.7	Concluding remarks about Painlevé's paradoxes	192
5.7.1	On equations of systems with Coulomb friction	192
5.7.2	On conditions of the paradoxes	193
5.7.3	On the reasons for the paradoxes	193
5.7.4	On the laws of motion in the paradoxical situations	193
5.7.5	On the initial motion of an immovable contact	194
5.7.6	On self-braking	194
5.7.7	On the mathematical description of Painlevé's law	195
5.7.8	On examples	195
6	Experimental investigations into the force of friction under self-excited oscillations	197
6.1	Experimental setups	198
6.1.1	The first setup	198
6.1.2	The second setup	200
6.1.3	The third setup	201

6.2	Determining the forces by means of an oscillogram	202
6.3	Change in the force of friction under break-down of the maximum friction in the case of a change in the velocity of motion	206
6.4	Dependence of the friction force on the rate of tangential loading	209
6.5	Plausibility of the dependence $F_+(f)$	213
6.5.1	Control tests	213
6.5.2	Estimating the numerical characteristics	213
6.5.3	Statistical properties of the dependences	214
6.5.4	Test data of other authors	215
6.6	Characteristic of the force of sliding friction	215
7	Force and small displacement in the contact	217
7.1	Components of the small displacement	217
7.1.1	Definition of break-down and initial break-down	217
7.1.2	Reversible and irreversible components	218
7.1.3	Influence of the intermediate stop and reverse on the irreversible displacement	220
7.1.4	Dependence of the total small displacement on the rate of tangential loading	222
7.1.5	Small displacement of parts of the contact	223
7.1.6	Comparing the values of small displacement with existing data	225
7.2	Remarks on friction between steel and polyamide	226
7.2.1	On critical values of the force of friction	226
7.2.2	Time lag of small displacement	226
7.2.3	Immovable and viscous components of the force of friction	229
7.3	Conclusions	230
8	Frictional self-excited oscillations	231
8.1	Self-excited oscillations due to hard excitation	231
8.1.1	The case of no structural damping	231
8.1.2	Including damping	237
8.2	Self-excited oscillations under both hard and soft excitations	240
8.2.1	Equations of motion	240
8.2.2	Critical velocities	242
8.2.3	Amplitude of auto-oscillation	244
8.2.4	Period of auto-oscillation	246
8.2.5	Self-excitation of systems	247
8.3	Accuracy of the displacement	249
	References	255
	Index	268