Contents

1. Introduction 1

2. Fractals 5
2.1 Fractal Structures 5
2.2 Fractal Dimensions 7
2.3 Methods for Obtaining Fractal Dimensions 10
2.4 Fractal Dimension of Aerogels 13
2.5 Brownian Motion and its Fractal Nature 15

3. Percolating Networks as Random Fractals 19
3.1 Critical Exponents and Scaling Relations 19
3.2 Fractal Dimension 27
3.3 Finite-Size Scaling and Scaling Relations 29
3.4 Nodes–Links–Blobs Model 32

4. Multifractals 35
4.1 Hierarchical Resistor Network Model 35
4.2 Mass Exponent and Generalized Dimension 40
4.3 Multifractal Spectrum 44
4.4 Relation between $\tau(q)$ and $f(\alpha)$ 45
4.5 Direct Determination of $f(\alpha)$ 47
4.6 Correlations between Box Measures 48
4.7 Profiles of $\tau(q)$, D_q, $f(\alpha)$, and $z(q)$ 50
4.8 Parabolic Approximation and Distribution Functions of Measures 54
4.9 Growth Probability of DLA 56

5. Anomalous Diffusion on Fractal Networks 59
5.1 Anomalous Diffusion 59
5.2 Spectral Dimension 62
5.3 Spectral Density of States of Fractal Networks 64
5.4 Scaling Argument for Spectral Density of States 65
5.5 Localization of Excitations on Fractal Networks 66
5.6 Phonons and Fractons in Percolating Networks 68

6. Atomic Vibrations of Percolating Networks 71
6.1 Spectral Density of States and Spectral Dimension 71
6.2 Missing Modes at Low Frequencies 75
6.3 The Hump at High Frequencies 76
6.4 Localized Nature of Fracton Excitations 79
6.5 Networks with Vector Elasticity: Scaling Arguments 81
6.6 Simulation Results for Vector Elasticity 86

7. Scaling Arguments for Dynamic Structure Factors 89
7.1 Dynamic Structure Factors: Inelastic Neutron Scattering 89
7.2 Single-Length Scaling Arguments 91
7.3 Numerical Simulations of $S(q, \omega)$ 94
7.4 Inelastic Light Scattering in Percolating Systems 96

8. Spin Waves in Diluted Heisenberg Antiferromagnets 101
8.1 Spin Waves in Percolating Antiferromagnets 101
8.2 Antiferromagnetic Spectral Dimension and the Upper Bound of $\tilde{d}_{\mathrm{AF}}$ 104
8.3 Numerical Simulations of Antiferromagnetic Fractons 107
8.4 Scaling Theory of $S(q, \omega)$ for Percolating Antiferromagnets 109
8.5 Large-Scale Simulations for $S(q, \omega)$ 110

9. Anderson Transition 115
9.1 Coherent Transport of Electrons 115
9.2 Anderson Localization 117
9.3 Scaling Theory of the Anderson Transition 120
9.4 Universality Classes 125
9.5 Numerical Studies 129
9.6 Dynamical Properties at the Anderson Transition 134
9.6.1 Scaling Form of Dynamic Quantities at Criticality 134
9.6.2 Diffusion of Wave Packets 135
9.6.3 Two-Particle Correlation Function 142
9.6.4 Diffusion Length 144

10. Multifractals in the Anderson Transition 149
10.1 Multifractality of a Critical Wavefunction 149
10.2 Multifractality of Spectral Measures 155
10.3 Relations between Multifractality and Dynamical Properties at the Transition Point 160
10.4 Conformal Invariance 162
10.5 Universality at Higher Landau Levels 168
10.6 Finite-Size Scaling for the q th Moment 172

Appendices 177
A. Multifractality of the HRN Model 177
B. Spectral Dimensions for Deterministic Fractals 182
B.1 Sierpinski Gasket 182
B.2 Mandelbrot–Given Fractal 184
C. Diffusion and Dynamics on Networks 186
C.1 Atomic Vibrations 187
C.2 Spin Waves in Diluted Ferro- and Antiferromagnets 191
C.3 Superconducting Networks 192
D. Wigner Distributions 194

References 197

Subject Index 203