
Contents

1	Introduction	1
1.1	What Is Complexity Theory?	1
1.2	Didactic Background	5
1.3	Overview	6
1.4	Additional Literature	10
2	Algorithmic Problems & Their Complexity	11
2.1	What Are Algorithmic Problems?	11
2.2	Some Important Algorithmic Problems	13
2.3	Measuring Computation Time	18
2.4	The Complexity of Algorithmic Problems	22
3	Fundamental Complexity Classes	25
3.1	The Special Role of Polynomial Computation Time	25
3.2	Randomized Algorithms	27
3.3	The Fundamental Complexity Classes for Algorithmic Problems	30
3.4	The Fundamental Complexity Classes for Decision Problems ..	35
3.5	Nondeterminism as a Special Case of Randomization	39
4	Reductions – Algorithmic Relationships Between Problems	43
4.1	When Are Two Problems Algorithmically Similar?	43
4.2	Reductions Between Various Variants of a Problem	46
4.3	Reductions Between Related Problems	49
4.4	Reductions Between Unrelated Problems	53
4.5	The Special Role of Polynomial Reductions	60
5	The Theory of NP-Completeness	63
5.1	Fundamental Considerations	63
5.2	Problems in NP	67
5.3	Alternative Characterizations of NP	69
5.4	Cook’s Theorem	70

6	NP-complete and NP-equivalent Problems	77
6.1	Fundamental Considerations	77
6.2	Traveling Salesperson Problems	77
6.3	Knapsack Problems	78
6.4	Partitioning and Scheduling Problems	80
6.5	Clique Problems	81
6.6	Team Building Problems	83
6.7	Championship Problems	85
7	The Complexity Analysis of Problems	89
7.1	The Dividing Line Between Easy and Hard	89
7.2	Pseudo-polynomial Algorithms and Strong NP-completeness . .	93
7.3	An Overview of the NP-completeness Proofs Considered	96
8	The Complexity of Approximation Problems – Classical Results	99
8.1	Complexity Classes	99
8.2	Approximation Algorithms	103
8.3	The Gap Technique	106
8.4	Approximation-Preserving Reductions	109
8.5	Complete Approximation Problems	112
9	The Complexity of Black Box Problems	115
9.1	Black Box Optimization	115
9.2	Yao’s Minimax Principle	118
9.3	Lower Bounds for Black Box Complexity	120
10	Additional Complexity Classes	127
10.1	Fundamental Considerations	127
10.2	Complexity Classes Within NP and co-NP	128
10.3	Oracle Classes	130
10.4	The Polynomial Hierarchy	132
10.5	BPP, NP, and the Polynomial Hierarchy	138
11	Interactive Proofs	145
11.1	Fundamental Considerations	145
11.2	Interactive Proof Systems	147
11.3	Regarding the Complexity of Graph Isomorphism Problems . .	148
11.4	Zero-Knowledge Proofs	155
12	The PCP Theorem and the Complexity of Approximation Problems	161
12.1	Randomized Verification of Proofs	161
12.2	The PCP Theorem	164
12.3	The PCP Theorem and Inapproximability Results	173
12.4	The PCP Theorem and APX-Completeness	177

13 Further Topics From Classical Complexity Theory	185
13.1 Overview	185
13.2 Space-Bounded Complexity Classes	186
13.3 PSPACE-complete Problems	188
13.4 Nondeterminism and Determinism in the Context of Bounded Space	191
13.5 Nondeterminism and Complementation with Precise Space Bounds	193
13.6 Complexity Classes Within P	195
13.7 The Complexity of Counting Problems	198
14 The Complexity of Non-uniform Problems	201
14.1 Fundamental Considerations	201
14.2 The Simulation of Turing Machines By Circuits	204
14.3 The Simulation of Circuits by Non-uniform Turing Machines	206
14.4 Branching Programs and Space Bounds	209
14.5 Polynomial Circuits for Problems in BPP	211
14.6 Complexity Classes for Computation with Help	212
14.7 Are There Polynomial Circuits for all Problems in NP?	214
15 Communication Complexity	219
15.1 The Communication Game	219
15.2 Lower Bounds for Communication Complexity	223
15.3 Nondeterministic Communication Protocols	233
15.4 Randomized Communication Protocols	238
15.5 Communication Complexity and VLSI Circuits	246
15.6 Communication Complexity and Computation Time	247
16 The Complexity of Boolean Functions	251
16.1 Fundamental Considerations	251
16.2 Circuit Size	252
16.3 Circuit Depth	254
16.4 The Size of Depth-Bounded Circuits	259
16.5 The Size of Depth-Bounded Threshold Circuits	264
16.6 The Size of Branching Programs	267
16.7 Reduction Notions	271
Final Comments	277
A Appendix	279
A.1 Orders of Magnitude and O -Notation	279
A.2 Results from Probability Theory	283
References	295
Index	301