

# Contents

<b>1</b>	<b>Grade Data Analysis — A First Look</b>	<b>1</b>
	<i>F. Ruland</i>	
1.1	”Questions” from clients . . . . .	1
1.2	About ”Grade Models and Methods for Data Analysis” . . . . .	2
1.3	Addressing the practitioner . . . . .	3
1.4	Addressing the theorist . . . . .	5
1.5	Regarding the analysis of data populations . . . . .	6
1.6	Overview of Grade Data Analysis algorithms . . . . .	7
1.7	Returning to the clients from the first page . . . . .	9
1.8	Conclusion — Chapter 1 . . . . .	11
<b>2</b>	<b>The Grade Approach</b>	<b>13</b>
	<i>F. Ruland</i>	
2.1	Introduction . . . . .	13
2.2	Part 1: Quick start to the understanding of grade concepts . . . . .	14
2.2.1	A simplified case of the grade approach . . . . .	14
2.2.2	Examples of data distribution sources . . . . .	15
2.3	Steps to making a concentration curve . . . . .	19
2.4	Quick Start summary . . . . .	27
2.5	Preview of Part 2, and suggestions before your eventual study of the multivariate material . . . . .	28
2.6	Part 2: Understanding concentration curves . . . . .	29
2.6.1	Introduction . . . . .	29
2.6.2	Two identical distributions . . . . .	31
2.6.3	Cylinder with partitions: cells of equal length, gas in equal proportions . . . . .	33
2.6.4	Constructing a concentration curve from individual cat- egory segments . . . . .	35
2.6.5	When proportions <i>do not</i> correspond between distri- butions . . . . .	36
2.6.6	Using the concentration curve to introduce the concept of overrepresentation . . . . .	37
2.6.7	Overrepresentation . . . . .	38
2.6.8	When we manipulate both distributions: gas (unequal proportions) and cylinder (unequal cell sizes) . . . . .	41
2.6.9	Example application — Winners versus losers in the car sales market . . . . .	43

2.6.10	Example application — Historic perspective (then vs. now) of car sales market . . . . .	45
2.6.11	Reordering (prioritizing) categories — and an introduction to the maximal concentration index . . . . .	47
2.6.12	Part 2 summary . . . . .	49
2.7	Chapter Summary . . . . .	50
<b>3</b>	<b>Univariate Lilliputian Model I</b>	<b>51</b>
	<i>T. Kowalczyk, W. Szczesny</i>	
3.1	Introduction . . . . .	51
3.2	Lilliputian variables and their basic parameters . . . . .	52
3.2.1	The cdf of a Lilliputian variable . . . . .	52
3.2.2	The expectation of a Lilliputian variable and the index <i>ar</i> . . . . .	56
3.2.3	The first moment Lilliputian variable, its variance, and the Gini Index . . . . .	61
3.2.4	Discontinuity measures . . . . .	67
3.3	The main equivalence relation which creates the Univariate Lilliputian Model . . . . .	70
3.3.1	Preliminary definitions and examples . . . . .	70
3.3.2	Equivalent pairs of random variables . . . . .	75
3.3.3	Grade transformations of univariate distributions . . . . .	77
3.4	Grade parameters . . . . .	80
3.4.1	The parameter <i>ar</i> . . . . .	80
3.4.2	Normal concentration pattern . . . . .	82
3.4.3	Likelihood ratio and local concentration . . . . .	85
3.5	Appendix . . . . .	87
3.5.1	Monotone grade probability transition function . . . . .	87
3.5.2	Properties of concentration measures . . . . .	89
<b>4</b>	<b>Univariate Lilliputian Model II</b>	<b>91</b>
	<i>T. Kowalczyk, E. Pleszczyńska, W. Szczesny</i>	
4.1	Introduction . . . . .	91
4.2	Lorenz Curve and Gini Index . . . . .	94
4.2.1	Ratio variables and related concentration curves . . . . .	94
4.2.2	First moment distribution and Lorenz curve . . . . .	101
4.2.3	Lorenz Curves with horizontal and/or vertical segments . . . . .	104
4.2.4	The variable called overrepresentation and its Lorenz curve . . . . .	106
4.2.5	Diagram of over- and underrepresentation . . . . .	111
4.2.6	Lorenz Curve and Gini Index for density transform of categorical variables . . . . .	115
4.3	Order oriented concentration curves . . . . .	116
4.3.1	Basic definitions . . . . .	116
4.3.2	The maximal concentration curve and the maximal concentration index . . . . .	120

4.3.3	Order oriented Lorenz Curve and inequality (Gini) index	122
4.3.4	Order oriented Lorenz Curve and Gini Index for the density transforms of categorical variables . . . . .	123
4.3.5	Link with the two-class discriminant analysis . . . . .	124
4.4	Dual concentration curve . . . . .	126
4.4.1	Definition of the dual concentration curve and dual Lorenz curve . . . . .	126
4.4.2	Random variable dual to a ratio variable . . . . .	129
4.4.3	Dual links between overrepresentation and underrepresentation . . . . .	130
4.4.4	Towards advantage problems in interpopulation comparisons . . . . .	132
4.5	Appendix . . . . .	133
4.5.1	Measurement scales . . . . .	133
4.5.2	Supplement to Section 4.2 (the inequality measures) . .	135
4.5.3	Supplement to Section 4.3.2 (the maximal concentration measures) . . . . .	136
4.5.4	Supplement to Section 4.3.3 (the ordered Lorenz Curve and Gini Index) . . . . .	136
4.5.5	Supplement to Section 4.4.2 (the random variable dual to a ratio variable) . . . . .	137
4.5.6	Bibliographical remarks to Chapter 3 and 4 . . . . .	137
<b>5</b>	<b>Asymmetry and the inverse concentration set</b>	<b>139</b>
	<i>A. Ciok</i>	
5.1	Introduction . . . . .	139
5.2	Concentration curves with a common value of the concentration index . . . . .	140
5.3	Links between asymmetry and opposite orderings . . . . .	145
5.4	Asymmetry in the Univariate Lilliputian Model . . . . .	146
5.4.1	Asymmetry curves . . . . .	146
5.4.2	Asymmetry index . . . . .	152
5.4.3	Families of curves with special properties . . . . .	153
5.5	Relative asymmetry . . . . .	154
5.5.1	Links with measurement scales . . . . .	154
5.5.2	Relative asymmetry measures . . . . .	155
5.5.3	Examples . . . . .	157
5.6	Appendix . . . . .	162
5.6.1	The inverse concentration set . . . . .	162
5.6.2	Asymmetry indices . . . . .	163
5.6.3	Bibliographical remarks . . . . .	165

<b>6</b>	<b>Discretization and regularity</b>	<b>167</b>
	<i>A. Ciok</i>	
6.1	Introduction . . . . .	167
6.2	Discretization framework . . . . .	168
6.3	Optimal discretization for a given number of categories . . . . .	170
6.4	Ideally regular concentration curves . . . . .	172
6.5	On the determination of the number of categories . . . . .	175
6.6	A parametric family of ideally regular Lilliputian curves . . . . .	178
6.7	Appendix . . . . .	181
	6.7.1 Optimal discretization . . . . .	181
	6.7.2 Algorithm of optimal discretization . . . . .	183
	6.7.3 Bibliographical remarks . . . . .	184
<b>7</b>	<b>Preliminary concepts of bivariate dependence</b>	<b>185</b>
	<i>T. Kowalczyk and W. Szczesny</i>	
7.1	Introduction . . . . .	185
7.2	Contingency tables with $m$ rows and $k$ columns . . . . .	185
7.3	Quadrant dependence . . . . .	190
7.4	Matrices of $ar$ 's for pairs of profiles. Total positivity of order two . . . . .	197
7.5	The regression function . . . . .	205
7.6	The monotone dependence function and the Gini Index . . . . .	210
7.7	Appendix - Bibliographical remarks . . . . .	214
<b>8</b>	<b>Dependence Lilliputian Model</b>	<b>217</b>
	<i>T. Kowalczyk, W. Szczesny, W. Wysocki</i>	
8.1	Introduction . . . . .	217
8.2	Grade bivariate distributions and overrepresentation maps for probability tables . . . . .	218
8.3	Lilliputian surfaces with uniform marginal distributions . . . . .	226
8.4	Spearman's $\rho$ and Kendall's $\tau$ expressed by volumes and masses in the unit cube . . . . .	234
8.5	Grade regression functions and related measures . . . . .	242
8.6	On permuting rows and columns of $m \times k$ probability tables . . . . .	246
	8.6.1 Maximal grade correlation . . . . .	246
	8.6.2 Ordered Gini indices for marginal density transforms . . . . .	250
	8.6.3 Maximal Kendall's $\tau$ . . . . .	256
8.7	The hinged sequences of rows and columns . . . . .	261
8.8	Appendix: Bibliographical remarks . . . . .	265
<b>9</b>	<b>Grade Correspondence Analysis and outlier detection</b>	<b>267</b>
	<i>O. Matyja, W. Szczesny</i>	
9.1	Introduction . . . . .	267
9.2	Algorithms of GCA . . . . .	268
	9.2.1 GCA algorithm based on Spearman's $\rho^*$ . . . . .	268
	9.2.2 GCA algorithm based on Kendall's $\tau$ . . . . .	272

9.2.3	GCA algorithm based on $\tau_{sgn}$ . . . . .	274
9.2.4	GCA and a mixture of permuted discretized binormal tables . . . . .	274
9.2.5	Folds . . . . .	277
9.3	Algorithm for Smooth Grade Correspondence Analysis (SGCA)	280
9.4	Examples of GCA and SGCA results . . . . .	282
9.4.1	A mixture of binormals . . . . .	282
9.4.2	BRIT <sub>7×7</sub> and CARS <sub>16×16</sub> . . . . .	283
9.5	Detection of rows and columns outlying the main trend . . . . .	286
9.5.1	Scatterplots for rows and for columns . . . . .	286
9.5.2	Measures of departure from TP <sub>2</sub> . . . . .	288
9.5.3	Rejecting outlying rows and columns . . . . .	291
9.6	Appendix - Bibliographical remarks . . . . .	295
<b>10</b>	<b>Cluster analysis based on GCA</b>	<b>297</b>
	<i>A. Ciok</i>	
10.1	Introduction . . . . .	297
10.2	Single and double grade clustering . . . . .	299
10.3	Optimal grade clustering . . . . .	305
10.4	Cluster analysis in the detection of mixtures . . . . .	307
10.4.1	Straight and reverse regular structures . . . . .	307
10.4.2	Survey of small business servicing firms . . . . .	310
10.4.3	SGCL results for the whole sample . . . . .	310
10.4.4	SGCL results for the particular branches . . . . .	312
10.4.5	Some final remarks . . . . .	314
10.5	Cluster analysis and the detection of an imprecisely defined trend	315
10.5.1	The use of sources of capital by retail trade firms in Poland . . . . .	315
10.5.2	Typology of firms for the pooled, three-year data . . . . .	315
10.5.3	Firm typologies for annual data . . . . .	318
10.5.4	Relationship between the generated firm typology and the firm profitability . . . . .	319
10.6	On GCCA application to various data sets . . . . .	321
10.7	Appendix . . . . .	323
10.7.1	An algorithm for optimal clustering . . . . .	323
10.7.2	Bibliographical remarks . . . . .	324
<b>11</b>	<b>Regularity and the number of clusters</b>	<b>325</b>
	<i>A. Ciok</i>	
11.1	Introduction . . . . .	325
11.2	Generalization of the parabola family from the ULM . . . . .	325
11.3	The ideal regularity of two-way data tables . . . . .	330
11.4	Regularity and cluster detection . . . . .	332
11.5	Cluster detection in finite data tables . . . . .	335
11.6	Appendix - Bibliographical remarks . . . . .	338

<b>12</b>	<b>Grade approach to the analysis of finite data matrices</b>	<b>339</b>
	<i>W. Szczesny</i>	
12.1	Introduction . . . . .	339
12.2	Insight Examples . . . . .	342
	12.2.1 The Competitors-Judges Data (C/J Example) . . . . .	343
	12.2.2 The Annual Bonus Data (A/B Example) . . . . .	349
12.3	Applicability of GCA . . . . .	353
12.4	A revisit of the univariate data . . . . .	356
12.5	Finite multivariate datasets and related inequality measures . . . . .	361
	12.5.1 Finite data tables and their grade regression functions . . . . .	361
	12.5.2 Lorenz Surfaces . . . . .	366
	12.5.3 Global differentiation and its decomposition . . . . .	373
	12.5.4 Decomposition of $\mathbf{Dif}_X$ . . . . .	377
12.6	Transformations of variables . . . . .	380
12.7	Detection of outliers and decomposition of a dataset . . . . .	381
<b>13</b>	<b>Inequality measures for multivariate distributions</b>	<b>385</b>
	<i>W. Szczesny</i>	
13.1	Introduction . . . . .	385
13.2	Inequality measures for multivariate distributions with finite sets of records . . . . .	389
13.3	Inequality measures for multivariate distributions with non-finite sets of records . . . . .	394
13.4	Inequality measures for continuous bivariate distributions . . . . .	398
	13.4.1 A pair of independent uniform Lilliputian variables . . . . .	398
	13.4.2 A pair of functionally dependent Lilliputian variables . . . . .	405
	13.4.3 A family of $TP_2$ distributions from $\mathbb{B}LM$ . . . . .	406
	13.4.4 Grade binormal distributions . . . . .	409
13.5	Inequality measures for grade multinormal distributions . . . . .	411
13.6	Inequality measures for the Moran distributions . . . . .	419
13.7	Appendix - link between grade similarity and dissimilarity of two regularly dependent random variables . . . . .	422
<b>14</b>	<b>Case studies with multivariate data</b>	<b>425</b>
	<i>W. Szczesny, M. Grzegorek</i>	
14.1	Introduction . . . . .	425
14.2	Case Study 1 - Main Trend of Questionnaire Data . . . . .	426
	14.2.1 The Questionnaire . . . . .	426
	14.2.2 The goal of the analysis . . . . .	427
	14.2.3 The Overrepresentation Map for Main Trend in dataset TOTAL . . . . .	428
	14.2.4 Interpretation of the results (with some general hints) . . . . .	430
14.3	Case Study 1 - Decomposition of the dataset into regular sub-populations . . . . .	432
	14.3.1 The Overrepresentation Maps for FIT-MT and OUT-MT433	

14.3.2	The grade strip charts for FIT-MT and OUT-MT . . .	434
14.3.3	Two-way ordered clustering . . . . .	436
14.4	Case Study 2 - Analysis of Engineering Data (Strength of Concrete) . . . . .	436
14.4.1	The variables: . . . . .	437
14.4.2	The goal of the analysis . . . . .	438
14.4.3	The Overrepresentation Map for Main Trend in the dataset TOTAL . . . . .	438
14.5	Case Study 2 - Decomposition of concrete mixtures into FIT-MT and OUT-MT . . . . .	439
14.5.1	The Overrepresentation Maps for FIT-MT and OUT-MT	440
14.5.2	The grade strip charts for FIT-MT and OUT-MT . . .	441
14.6	Final remarks for the two case studies . . . . .	444
14.7	Appendix: . . . . .	444
14.7.1	Case Study 1 - further details of the analysis . . . . .	444
14.7.2	Case Study 2 - further details of the analysis . . . . .	448
14.7.3	Bibliographical remarks . . . . .	453
<b>15</b>	<b>The GradeStat program</b>	<b>455</b>
	<i>O. Matyja</i>	
15.1	Introduction . . . . .	455
15.2	Main implemented features . . . . .	455
15.2.1	Data overview . . . . .	455
15.2.2	Charts . . . . .	456
15.2.3	Preprocessing . . . . .	456
15.2.4	Ordering . . . . .	457
15.2.5	Clustering . . . . .	457
	<b>References</b>	<b>459</b>
	<b>Index</b>	<b>468</b>