Contents

1 Introduction 1

2 Connected Reductive Groups and Their Lie Algebras 5
2.1 Notation and Background 5
2.1.1 H-Varieties and Adjoint Action of H on $\mathcal{H}$ 6
2.1.4 Reductive Groups 7
2.1.10 About Intersections of Lie Algebras of Closed Subgroups of G 9
2.1.16 $\mathbb{F}_q$-Structures 10
2.2 Chevalley Formulas 11
2.3 The Lie Algebra of Z_G 13
2.4 Existence of Chevalley Bases on $\mathcal{G}'$ 15
2.5 Existence of Non-degenerate G-Invariant Bilinear Forms on $\mathcal{G}$. 18
2.6 Centralizers 24
2.7 The Varieties G_{uni} and $\mathcal{G}_{nil}$ 30

3 Deligne-Lusztig Induction 33
3.1 The Space of G^F-Invariant Functions on $\mathcal{G}^F$ 33
3.2 Deligne-Lusztig Induction: Definition and Basic Properties 36
3.2.1 Deligne-Lusztig Induction: The Group Case 36
3.2.8 Deligne-Lusztig Induction: The Lie Algebra Case 38
3.2.17 Basic Properties of $\mathcal{R}^{\mathcal{G}}_{\mathcal{L}\subset\mathcal{P}}$ 40

4 Local Systems and Perverse Sheaves 45
4.1 Simple Perverse Sheaves, Intersection Cohomology Complexes . 47
4.2 H-Equivariance 49
4.3 Locally (Iso)trivial Principal H-Bundles 54
4.4 F-Equivariant Sheaves and Complexes 57

5 Geometrical Induction 61
5.1 Admissible Complexes and Orbital Perverse Sheaves on $\mathcal{G}$ 62
5.1.1 Parabolic Induction of Equivariant Perverse Sheaves . . . 63
5.1.9 The Complexes $\mathrm{ind}_{\mathcal{L}\subset\mathcal{P}}^{\mathcal{G}} K(\Sigma,\mathcal{E})$ 64
5.1.14 The Complexes $\mathrm{ind}_{\mathcal{L}\subset\mathcal{P}}^{\mathcal{G}} K(\Sigma,\mathcal{E})$ Are G-Equivariant Perverse Sheaves 66
5.1.26 When the Complexes $\mathrm{ind}_{\mathcal{L}\subset\mathcal{P}}^{\mathcal{G}} K(\Sigma,\mathcal{E})$ Are Intersection Cohomology Complexes 73
5.1.41 Restriction of $\mathrm{ind}_{\mathcal{L}\subset\mathcal{P}}^{\mathcal{G}} K(\Sigma,\mathcal{E})$ to $\mathcal{G}_\sigma$ with $\sigma \in z(\mathcal{G})$ 79
5.1.51 Introducing Frobenius 81
5.1.56 Admissible Complexes (or Character Sheaves) on $\mathcal{G}$ 84
5.1.72 Orbital Perverse Sheaves: The Fundamental Theorem . . 86
5.2 Deligne-Fourier Transforms and Admissible Complexes 89
5.3 Endomorphism Algebra of Lusztig Complexes 96
5.4 Geometrical Induction: Definition 99
5.4.1 Preliminaries 100
5.4.10 Geometrical Induction 103
5.5 Deligne-Lusztig Induction and Geometrical Induction 106
5.5.1 Generalized Green Functions 106
5.5.9 The Character Formula 110
5.5.11 Generalized Green Functions and Two-Variable Green Functions 111
5.5.14 Geometrical Induction and Deligne-Lusztig Induction . . 112

6 Deligne-Lusztig Induction and Fourier Transforms 115
6.1 Frobenius Action on the Parabolic Induction of Cuspidal Orbital Perverse Sheaves 115
6.1.1 The Functor $\mathrm{ind}_{S\times\mathcal{L},\mathcal{P}}^{S\times\mathcal{G}} : \mathcal{M}_L(S\times\mathcal{L}) \to \mathcal{D}_c^b(S\times\mathcal{G})$ 116
6.1.2 The Complexes $\mathrm{ind}_{S\times\mathcal{L},\mathcal{P}}^{S\times\mathcal{G}} K(\mathcal{Z}\times C,\mathcal{E})$ 116
6.1.15 The Complexes K_1 and K_2 121
6.1.19 The Character Formula 122
6.1.54 Deligne-Lusztig Induction and Geometrical Induction . . 137
6.2 On the Conjecture 3.2.30 139
6.2.1 Reduction of 3.2.30 to the Case of Nilpotently Supported Cuspidal Functions 139
6.2.7 The Main Results 142
6.2.20 Lusztig Constants: A Formula 146

7 Fourier Transforms of the Characteristic Functions of the Adjoint Orbits 151
7.1 Preliminaries 151
7.1.1 A Decomposition of $\mathcal{C}(\mathcal{G}^F)$ 151
7.1.6 A Geometric Analogue of 3.2.24 153
7.2 Fourier Transforms of the Characteristic Functions of the Adjoint Orbits 154
7.3 Fourier Transforms of the Characteristic Functions of the Semi-simple Orbits 157

References 159

Index 163