
Contents

Introduction	1
1 Four Hamiltonian Systems	9
1.1 Small Vibrations of Tetrahedral Molecules	9
1.1.1 Description	9
1.1.2 The 2-Mode	11
1.1.3 The 3-Mode	16
1.2 The Hydrogen Atom in Crossed Fields	17
1.2.1 Perturbed Kepler Systems	17
1.2.2 Description	18
1.2.3 Normalization and Reduction	19
1.2.4 Energy Momentum Map	20
1.3 Quadratic Spherical Pendula	22
1.3.1 A Spherical Pendulum Model for Floppy Triatomic Molecules	22
1.3.2 The Family of Quadratic Spherical Pendula	23
1.4 The 1: - 2 Resonance System	26
1.4.1 Reduction	27
1.4.2 The 1: - 1 Resonance System	30
1.4.3 Fractional Monodromy in the 1: - 2 Resonance System ..	30
2 Small Vibrations of Tetrahedral Molecules	35
2.1 Discrete and Continuous Symmetry	35
2.1.1 The Hamiltonian Family	35
2.1.2 Dynamical Symmetry. Relative Equilibria	37
2.1.3 Symmetry and Topology	40
2.2 One-Parameter Classification	43
2.3 Normalization and Reduction	46
2.4 Relative Equilibria Corresponding to Critical Points	47
2.5 Relative Equilibria Corresponding to Non-critical Points	51

2.5.1	Existence and Stability of the $C_s \wedge T_2$ Relative Equilibria	51
2.5.2	Configuration Space Image of the $C_s \wedge T_2$ Relative Equilibria	54
2.6	Bifurcations	56
2.7	The 3-Mode as a 3-DOF Analogue of the Hénon-Heiles Hamiltonian	57
3	The Hydrogen Atom in Crossed Fields	59
3.1	Review of the Keplerian Normalization	59
3.1.1	Kustaanheimo-Stiefel Regularization	59
3.1.2	First Normalization	60
3.1.3	First Reduction	61
3.2	Second Normalization and Reduction	63
3.2.1	Second Normalization	63
3.2.2	Second Reduction	64
3.2.3	Fixed Points	66
3.3	Discrete Symmetries and Reconstruction	66
3.4	The Hamiltonian Hopf Bifurcations	68
3.4.1	Local Chart	69
3.4.2	Flattening of the Symplectic Form	70
3.4.3	S^1 Symmetry	71
3.4.4	Linear Hamiltonian Hopf Bifurcation	72
3.4.5	Nonlinear Hamiltonian Hopf Bifurcation	75
3.5	Hamiltonian Hopf Bifurcation and Monodromy	77
3.6	Description of the Hamiltonian Hopf Bifurcation on the Fully Reduced Space	81
3.6.1	The Standard Situation	81
3.6.2	The Hydrogen Atom in Crossed Fields	82
3.6.3	Degeneracy	85
4	Quadratic Spherical Pendula	87
4.1	Generalities	87
4.1.1	Constrained Equations of Motion	87
4.1.2	Reduction of the Axial Symmetry	90
4.2	Classification of Quadratic Spherical Pendula	91
4.2.1	Critical Values of the Energy-Momentum Map	91
4.2.2	Reconstruction	94
4.3	Classical and Quantum Monodromy	98
4.3.1	Classical Monodromy	98
4.3.2	Quantum Monodromy	100
4.4	Monodromy in the Family of Quadratic Spherical Pendula	101
4.4.1	Monodromy in Type O and Type II Systems	102
4.4.2	Non-local Monodromy	103
4.5	Quantum Monodromy in the Quadratic Spherical Pendula	104

4.6	Geometric Hamiltonian Hopf Bifurcations	106
4.7	The LiCN Molecule	110
5	Fractional Monodromy in the 1: – 2 Resonance System	113
5.1	The Energy-Momentum Map	113
5.1.1	Reduction	114
5.1.2	The Discriminant Locus	114
5.1.3	Reconstruction	117
5.2	The Period Lattice Description of Fractional Monodromy	119
5.2.1	Rotation Angle and First Return Time	121
5.2.2	The Modified Period Lattice	122
5.3	Sketch of the Proof of Fractional Monodromy in [43]	124
5.4	Relation to the 1: – 2 Resonance System of [99]	125
5.5	Quantum Fractional Monodromy	126
5.6	Fractional Monodromy in Other Resonances	127
 Appendix		
A	The Tetrahedral Group	129
A.1	Action of the Group $T_d \times \mathcal{T}$ on the Spaces \mathbf{R}^3 and $T^*\mathbf{R}^3$	129
A.2	Fixed Points of the Action of $T_d \times \mathcal{T}$ on \mathbf{CP}^2	130
A.3	Subspaces of \mathbf{CP}^2 Invariant Under the Action of $T_d \times \mathcal{T}$	131
A.4	Action of $T_d \times \mathcal{T}$ on the Projections of Nonlinear Normal Modes in the Configuration Space \mathbf{R}^3	133
B	Local Properties of Equilibria	135
B.1	Stability of Equilibria	135
B.2	Morse Inequalities and the Euler Characteristic	136
B.3	Linearization Near Equilibria on \mathbf{CP}^2	137
References		139
Index		147