
Contents

1	Introduction	1
1.1	Usefulness of Numerical Investigations	1
1.2	Development of Numerical Methods	4
1.3	Characterization of Numerical Methods	6
2	Modeling of Continuum Mechanical Problems	11
2.1	Kinematics	11
2.2	Basic Conservation Equations	15
2.2.1	Mass Conservation	16
2.2.2	Momentum Conservation	18
2.2.3	Moment of Momentum Conservation	19
2.2.4	Energy Conservation	19
2.2.5	Material Laws	20
2.3	Scalar Problems	20
2.3.1	Simple Field Problems	21
2.3.2	Heat Transfer Problems	23
2.4	Structural Mechanics Problems	26
2.4.1	Linear Elasticity	27
2.4.2	Bars and Beams	30
2.4.3	Disks and Plates	35
2.4.4	Linear Thermo-Elasticity	39
2.4.5	Hyperelasticity	40
2.5	Fluid Mechanical Problems	42
2.5.1	Incompressible Flows	43
2.5.2	Inviscid Flows	45
2.6	Coupled Fluid-Solid Problems	46
2.6.1	Modeling	47
2.6.2	Examples of applications	49
	Exercises for Chap. 2	56

3	Discretization of Problem Domain	57
3.1	Description of Problem Geometry	57
3.2	Numerical Grids	60
3.2.1	Grid Types	61
3.2.2	Grid Structure	62
3.3	Generation of Structured Grids	66
3.3.1	Algebraic Grid Generation	67
3.3.2	Elliptic Grid Generation	69
3.4	Generation of Unstructured Grids	71
3.4.1	Advancing Front Methods	72
3.4.2	Delaunay Triangulations	74
	Exercises for Chap. 3	76
4	Finite-Volume Methods	77
4.1	General Methodology	77
4.2	Approximation of Surface and Volume Integrals	81
4.3	Discretization of Convective Fluxes	84
4.3.1	Central Differences	85
4.3.2	Upwind Techniques	86
4.3.3	Flux-Blending Technique	88
4.4	Discretization of Diffusive Fluxes	89
4.5	Non-Cartesian Grids	91
4.6	Discrete Transport Equation	94
4.7	Treatment of Boundary Conditions	95
4.8	Algebraic System of Equations	97
4.9	Numerical Example	100
	Exercises for Chap. 4	103
5	Finite-Element Methods	107
5.1	Galerkin Method	107
5.2	Finite-Element Discretization	110
5.3	One-Dimensional Linear Elements	112
5.3.1	Discretization	112
5.3.2	Global and Local View	115
5.4	Practical Realization	118
5.4.1	Assembling of Equation Systems	118
5.4.2	Computation of Element Contributions	120
5.4.3	Numerical Example	121
5.5	One-Dimensional Cubic Elements	123
5.5.1	Discretization	123
5.5.2	Numerical Example	126
5.6	Two-Dimensional Elements	128
5.6.1	Variable Transformation for Triangular Elements	129
5.6.2	Linear Triangular Elements	131
5.6.3	Numerical Example	132

5.6.4	Bilinear Parallelogram Elements	138
5.6.5	Other Two-Dimensional Elements	140
5.7	Numerical Integration	143
	Exercises for Chap. 5	146
6	Time Discretization	149
6.1	Basics	149
6.2	Explicit Methods	154
6.3	Implicit Methods	157
6.4	Numerical Example	161
	Exercises for Chap. 6	165
7	Solution of Algebraic Systems of Equations	167
7.1	Linear Systems	167
7.1.1	Direct Solution Methods	168
7.1.2	Basic Iterative Methods	169
7.1.3	ILU Methods	171
7.1.4	Convergence of Iterative Methods	174
7.1.5	Conjugate Gradient Methods	176
7.1.6	Preconditioning	178
7.1.7	Comparison of Solution Methods	179
7.2	Non-Linear and Coupled Systems	182
	Exercises for Chap. 7	184
8	Properties of Numerical Methods	187
8.1	Properties of Discretization Methods	187
8.1.1	Consistency	188
8.1.2	Stability	191
8.1.3	Convergence	195
8.1.4	Conservativity	196
8.1.5	Boundedness	197
8.2	Estimation of Discretization Error	199
8.3	Influence of Numerical Grid	202
8.4	Cost Effectiveness	206
	Exercises for Chap. 8	206
9	Finite-Element Methods in Structural Mechanics	209
9.1	Structure of Equation System	209
9.2	Finite-Element Discretization	211
9.3	Examples of Applications	215
	Exercises for Chap. 9	221

10	Finite-Volume Methods for Incompressible Flows	223
10.1	Structure of Equation System	223
10.2	Finite-Volume Discretization	224
10.3	Solution Algorithms	230
10.3.1	Pressure-Correction Methods	231
10.3.2	Pressure-Velocity Coupling	235
10.3.3	Under-Relaxation	239
10.3.4	Pressure-Correction Variants	244
10.4	Treatment of Boundary Conditions	247
10.5	Example of Application	251
	Exercises for Chap. 10	258
11	Computation of Turbulent Flows	259
11.1	Characterization of Computational Methods	259
11.2	Statistical Turbulence Modeling	261
11.2.1	The k - ε Turbulence Model	263
11.2.2	Boundary Conditions	265
11.2.3	Discretization and Solution Methods	270
11.3	Large Eddy Simulation	271
11.4	Comparison of Approaches	275
12	Acceleration of Computations	277
12.1	Adaptivity	277
12.1.1	Refinement Strategies	278
12.1.2	Error Indicators	280
12.2	Multi-Grid Methods	281
12.2.1	Principle of Multi-Grid Method	282
12.2.2	Two-Grid Method	284
12.2.3	Grid Transfers	287
12.2.4	Multigrid Cycles	288
12.2.5	Examples of Computations	290
12.3	Parallelization of Computations	295
12.3.1	Parallel Computer Systems	296
12.3.2	Parallelization Strategies	297
12.3.3	Efficiency Considerations and Example Computations	302
	Exercises for Chap. 12	306
	List of Symbols	307
	References	313
	Index	317