
Contents

1	Lévy Processes and Applications	1
1.1	Lévy Processes and Infinite Divisibility	1
1.2	Some Examples of Lévy Processes	5
1.3	Lévy Processes and Some Applied Probability Models	14
	Exercises	26
2	The Lévy–Itô Decomposition and Path Structure	33
2.1	The Lévy–Itô Decomposition	33
2.2	Poisson Random Measures	35
2.3	Functionals of Poisson Random Measures	41
2.4	Square Integrable Martingales	44
2.5	Proof of the Lévy–Itô Decomposition	51
2.6	Lévy Processes Distinguished by Their Path Type	53
2.7	Interpretations of the Lévy–Itô Decomposition	56
	Exercises	62
3	More Distributional and Path-Related Properties	67
3.1	The Strong Markov Property	67
3.2	Duality	73
3.3	Exponential Moments and Martingales	75
	Exercises	83
4	General Storage Models and Paths of Bounded Variation ..	87
4.1	General Storage Models	87
4.2	Idle Times	88
4.3	Change of Variable and Compensation Formulae	90
4.4	The Kella–Whitt Martingale	97
4.5	Stationary Distribution of the Workload	100
4.6	Small-Time Behaviour and the Pollaczek–Khintchine Formula ..	102
	Exercises	105

5	Subordinators at First Passage and Renewal Measures	111
5.1	Killed Subordinators and Renewal Measures	111
5.2	Overshoots and Undershoots	119
5.3	Creeping	121
5.4	Regular Variation and Tauberian Theorems	126
5.5	Dynkin–Lamperti Asymptotics	130
	Exercises	133
6	The Wiener–Hopf Factorisation	139
6.1	Local Time at the Maximum	140
6.2	The Ladder Process	147
6.3	Excursions	154
6.4	The Wiener–Hopf Factorisation	157
6.5	Examples of the Wiener–Hopf Factorisation	168
6.6	Brief Remarks on the Term “Wiener–Hopf”	174
	Exercises	174
7	Lévy Processes at First Passage and Insurance Risk	179
7.1	Drifting and Oscillating	179
7.2	Cramér’s Estimate of Ruin	185
7.3	A Quintuple Law at First Passage	189
7.4	The Jump Measure of the Ascending Ladder Height Process . .	195
7.5	Creeping	197
7.6	Regular Variation and Infinite Divisibility	200
7.7	Asymptotic Ruinous Behaviour with Regular Variation	203
	Exercises	206
8	Exit Problems for Spectrally Negative Processes	211
8.1	Basic Properties Reviewed	211
8.2	The One-Sided and Two-Sided Exit Problems	214
8.3	The Scale Functions $W^{(q)}$ and $Z^{(q)}$	220
8.4	Potential Measures	223
8.5	Identities for Reflected Processes	227
8.6	Brief Remarks on Spectrally Negative GOUs	231
	Exercises	233
9	Applications to Optimal Stopping Problems	239
9.1	Sufficient Conditions for Optimality	240
9.2	The McKean Optimal Stopping Problem	241
9.3	Smooth Fit versus Continuous Fit	245
9.4	The Novikov–Shiryaev Optimal Stopping Problem	249
9.5	The Shepp–Shiryaev Optimal Stopping Problem	255
9.6	Stochastic Games	260
	Exercises	269

10 Continuous-State Branching Processes	271
10.1 The Lamperti Transform	271
10.2 Long-term Behaviour	274
10.3 Conditioned Processes and Immigration	280
10.4 Concluding Remarks	291
Exercises	293
Epilogue	295
Solutions	299
References	361
Index	371