

---

# Contents

<b>1</b>	<b>Preliminary results</b> .....	1
1.1	Notations .....	1
1.1.1	Definitions .....	1
1.1.2	Asymptotic notations .....	2
1.1.3	Hölder inequality .....	2
1.1.4	Young inequality .....	3
1.1.5	Sobolev imbedding inequality .....	3
1.1.6	An interpolation inequality .....	3
1.1.7	Contraction mapping principle .....	4
1.1.8	Gronwall's Lemma .....	4
1.2	Local existence .....	5
1.3	Small initial data .....	13
1.4	Large initial data .....	17
1.5	Estimates for linear semigroups .....	26
1.5.1	Estimates in weighted Lebesgue spaces .....	26
1.5.2	Estimates in the $\mathbf{L}^2$ - theory .....	37
1.5.3	Estimates in Fourier spaces .....	41
1.5.4	Estimates for large $x$ and $t$ .....	46
<b>2</b>	<b>Weak Nonlinearity</b> .....	51
2.1	General approach .....	51
2.2	Asymptotics for large $x$ and $t$ .....	66
2.2.1	Small initial data .....	67
2.2.2	Large initial data .....	74
2.2.3	Nonlinear heat equation .....	80
2.3	Damped wave equation .....	82
2.3.1	Large initial data .....	92
2.4	Sobolev type equations .....	109
2.4.1	Local existence .....	111
2.4.2	Small data .....	112
2.4.3	Large data .....	116

2.5	Whitham type equation	124
2.5.1	A model equation	124
2.5.2	Local existence and smoothing effect	125
2.5.3	Small initial data	133
2.5.4	Large initial data	140
2.6	Weak dissipation, strong dispersion	143
2.6.1	Preliminary Lemmas	146
2.6.2	Proof of Theorem 2.59	151
2.7	A system of nonlinear equations	157
2.7.1	Local existence and smoothing effect	160
2.7.2	Global existence and asymptotic behavior	163
2.7.3	Large initial data	169
2.8	Comments	175
<b>3</b>	<b>Critical Nonconvective Equations</b>	<b>179</b>
3.1	General approach	179
3.2	Fractional equations	194
3.2.1	Small data	194
3.2.2	Large data	198
3.3	Asymptotics for large $x$ and $t$	205
3.3.1	Small initial data	206
3.3.2	Large initial data	209
3.3.3	Nonlinear heat equation	210
3.4	Complex Landau-Ginzburg equation	218
3.4.1	Preliminaries	221
3.4.2	Proof of Theorem 3.22 for $\operatorname{Re} \delta(\alpha, \beta) > 0$	222
3.4.3	Proof of Theorem 3.22 for $\eta = 0, \mu > 0$	223
3.4.4	Proof of Theorem 3.22 for $\eta = 0, \mu = 0, \kappa > 0$	231
3.4.5	Asymptotic expansion	242
3.5	Damped wave equation	254
3.5.1	Small initial data	254
3.5.2	Large initial data	268
3.6	Sobolev type equations	286
3.6.1	Small initial data	287
3.6.2	Large initial data	290
3.7	Whitham type equations	298
3.7.1	Preliminary Lemmas	302
3.7.2	Proof of Theorem 3.56	316
3.8	Comments	319
<b>4</b>	<b>Critical Convective Equations</b>	<b>323</b>
4.1	General approach	323
4.2	Whitham equation	330
4.2.1	Preliminary Lemmas	333
4.2.2	Proof of Theorem 4.9	336

4.2.3	Self-similar solutions	339
4.2.4	Proof of Theorem 4.13	342
4.3	KdV-B equation	349
4.3.1	Lemmas	351
4.3.2	Proof of Theorem 4.18	367
4.3.3	Second term of asymptotics	372
4.3.4	Proof of Theorem 4.25	372
4.4	BBM-B equation	381
4.4.1	Preliminaries	382
4.4.2	Proof of Theorem 4.28	395
4.4.3	Proof of Theorem 4.29	398
4.5	A system of nonlinear equations	402
4.5.1	Preliminary Lemmas	407
4.5.2	Proof of Theorem 4.36	409
4.5.3	Self-similar solutions	411
4.5.4	Proof of Theorem 4.38	413
4.6	Comments	427
<b>5</b>	<b>Subcritical Nonconvective Equations</b>	<b>431</b>
5.1	General approach	431
5.2	Fractional heat equations	456
5.2.1	Small initial data	457
5.2.2	Large initial data	462
5.3	Whitham type equations	465
5.3.1	Preliminary Lemmas	468
5.3.2	Proof of Theorem 5.19	474
5.4	Damped wave equation	476
5.4.1	Small initial data	476
5.4.2	Large data	487
5.5	Sobolev type equations	491
5.5.1	Small data	492
5.5.2	Large data	494
5.6	Oscillating solutions to nonlinear heat equation	498
5.6.1	Lemmas	499
5.6.2	Proof of Theorem 5.37	506
5.7	Comments	510
<b>6</b>	<b>Subcritical Convective Equations</b>	<b>513</b>
6.1	Burgers type equations	513
6.1.1	Rarefaction wave	514
6.1.2	Shock wave	522
6.1.3	Zero boundary conditions	529
6.2	Comments	539
	<b>References</b>	<b>541</b>
	<b>Index</b>	<b>555</b>