Contents

Preface VII

1 Topological Foundations 1
1.1 Manifolds and Differentiable Manifolds 1
1.2 Homotopy of Maps. The Fundamental Group 3
1.3 Coverings 6
1.4 Global Continuation of Functions on Simply-Connected Manifolds 15

2 Differential Geometry of Riemann Surfaces 17
2.1 The Concept of a Riemann Surface 17
2.2 Some Simple Properties of Riemann Surfaces 19
2.3 Metrics on Riemann Surfaces 20
2.3.A Triangulations of Compact Riemann Surfaces 31
2.4 Discrete Groups of Hyperbolic Isometries. Fundamental Polygons. Some Basic Concepts of Surface Topology and Geometry. 39
2.4.A The Topological Classification of Compact Riemann Surfaces 54
2.5 The Theorems of Gauss-Bonnet and Riemann-Hurwitz 57
2.6 A General Schwarz Lemma 64
2.7 Conformal Structures on Tori 72

3 Harmonic Maps 79
3.1 Review: Banach and Hilbert Spaces. The Hilbert Space L^2 .. 79
3.2 The Sobolev Space $W^{1,2} = H^{1,2}$ 91
3.3 The Dirichlet Principle. Weak Solutions of the Poisson Equation 99
3.4 Harmonic and Subharmonic Functions 103
3.5 The C^α Regularity Theory 110
3.6 Maps Between Surfaces. The Energy Integral. Definition and Simple Properties of Harmonic Maps 119
3.7 Existence of Harmonic Maps 125
3.8 Regularity of Harmonic Maps 134
3.9 Uniqueness of Harmonic Maps 138
3.10 Harmonic Diffeomorphisms 144

3.11 Metrics and Conformal Structures 152

4 Teichmüller Spaces 161
4.1 The Basic Definitions 161
4.2 Harmonic Maps, Conformal Structures and Holomorphic Quadratic Differentials. Teichmüller's Theorem 163
4.3 Fenchel-Nielsen Coordinates. An Alternative Approach to the Topology of Teichmüller Space 173
4.4 Uniformization of Compact Riemann Surfaces 183

5 Geometric Structures on Riemann Surfaces 187
5.1 Preliminaries: Cohomology and Homology Groups 187
5.2 Harmonic and Holomorphic Differential Forms on Riemann Surfaces 195
5.3 The Periods of Holomorphic and Meromorphic Differential Forms 203
5.4 Divisors. The Riemann-Roch Theorem 208
5.5 Holomorphic 1-Forms and Metrics on Compact Riemann Surfaces 220
5.6 Divisors and Line Bundles 222
5.7 Projective Embeddings 233
5.8 Algebraic Curves 240
5.9 Abel's Theorem and the Jacobi Inversion Theorem 253
5.10 Elliptic Curves 260

Bibliography 269

Index of Notation 271

Index 273