
Contents

1	One-Dimensional, First-Order Systems	1
1.1	Linear Systems	2
1.1.1	Characterization of the Solution	2
1.1.2	Existence of Steady-State Equilibria	4
1.1.3	Uniqueness of Steady-State Equilibria	5
1.1.4	Stability of Steady-State Equilibria	6
1.2	Nonlinear Systems	16
1.2.1	The Solution	16
1.2.2	Existence, Uniqueness and Multiplicity of Steady-State Equilibria	17
1.2.3	Linearization and Local Stability of Steady-State Equilibria	19
1.2.4	Global Stability	24
2	Multi-Dimensional, First-Order, Linear Systems: Solution	27
2.1	Characterization of the Solution	28
2.2	Existence and Uniqueness of Steady-State Equilibria	30
2.3	Examples of Two-Dimensional Systems	33
2.3.1	Explicit Solution and Stability Analysis	33
2.3.2	Stability Analysis Without an Explicit Solution	46
2.4	Properties of the Jordan Matrix	51
2.5	Representation of the System in the Jordan Normal Form	56
2.5.1	Transformation of Non-Homogeneous Systems into Homogeneous Ones	56
2.5.2	The Solution in Terms of the Jordan Normal Form	57

3	Multi-Dimensional, First-Order, Linear Systems:	
	Characterization	59
3.1	Distinct Real Eigenvalues	59
	3.1.1 Characterization of the Solution	59
	3.1.2 Phase Diagrams of Two-Dimensional Uncoupled Systems	62
3.2	Repeated Real Eigenvalues	68
	3.2.1 Characterization of the Solution	68
	3.2.2 Phase Diagram of the Two-Dimensional Case	71
3.3	Distinct Pairs of Complex Eigenvalues	77
	3.3.1 Characterization of the Solution	77
	3.3.2 Phase Diagram of a Two-Dimensional System	82
3.4	Repeated Pairs of Complex Eigenvalues	84
3.5	The General Case	86
3.6	Characterization of Two-Dimensional Systems in Terms of $\text{tr}A$ and $\det A$	87
4	Multi-Dimensional, First-Order, Nonlinear Systems ..	93
4.1	Local Analysis	96
	4.1.1 Linearization	96
	4.1.2 Stable, Unstable, and Center Eigenspaces	98
	4.1.3 Local Stable and Unstable Manifolds	101
	4.1.4 The <i>Stable Manifold Theorem</i>	102
4.2	Global Analysis	104
5	Higher-Order and Non-Autonomous Systems	107
5.1	Higher-Order Systems	107
	5.1.1 Linear Systems	107
	5.1.2 Nonlinear Systems	111
5.2	Non-Autonomous Systems	112
6	Examples of Two-Dimensional Systems	115
6.1	First-Order Linear Systems	115
	6.1.1 Real, Distinct, Positive Eigenvalues	115
	6.1.2 Complex Eigenvalues - Periodic Orbit	126
	6.1.3 Complex Eigenvalues - Spiral Sink	133
6.2	Second-Order Linear Systems	137
6.3	Nonlinear Systems	144
	Glossary	147
	References	149
	Index	151