

Contents

Introduction	1
Part I: Basics	
1 Fundamental Coefficients	5
1.1 Elementary Counting Principles	5
Exercises	9
1.2 Subsets and Binomial Coefficients	10
Exercises	18
1.3 Set-partitions and Stirling Numbers $S_{n,k}$	20
Exercises	23
1.4 Permutations and Stirling Numbers $s_{n,k}$	24
Exercises	29
1.5 Number-Partitions	31
Exercises	35
1.6 Lattice Paths and Gaussian Coefficients	36
Exercises	42
Highlight: Aztec Diamonds	44
Notes and References	51
2 Formal Series and Infinite Matrices	53
2.1 Algebra of Formal Series	53
Exercises	59
2.2 Types of Formal Series	60
Exercises	65
2.3 Infinite Sums and Products	66
Exercises	70
2.4 Infinite Matrices and Inversion of Sequences	71
Exercises	76
2.5 Probability Generating Functions	77
Exercises	84
Highlight: The Point of (No) Return	85
Notes and References	90

Part II: Methods

3	Generating Functions	93
3.1	Solving Recurrences	93
	Exercises	102
3.2	Evaluating Sums	105
	Exercises	110
3.3	The Exponential Formula	112
	Exercises	122
3.4	Number-Partitions and Infinite Products	124
	Exercises	132
	Highlight: Ramanujan's Most Beautiful Formula	136
	Notes and References	141
4	Hypergeometric Summation	143
4.1	Summation by Elimination	143
	Exercises	148
4.2	Indefinite Sums and Closed Forms	148
	Exercises	155
4.3	Recurrences for Hypergeometric Sums	155
	Exercises	161
4.4	Hypergeometric Series	162
	Exercises	168
	Highlight: New Identities from Old	171
	Notes and References	178
5	Sieve Methods	179
5.1	Inclusion-Exclusion	179
	Exercises	189
5.2	Möbius Inversion	191
	Exercises	200
5.3	The Involution Principle	202
	Exercises	215
5.4	The Lemma of Gessel-Viennot	217
	Exercises	229
	Highlight: Tutte's Matrix-Tree Theorem	231
	Notes and References	237

6 Enumeration of Patterns 239

 6.1 Symmetries and Patterns 239

 Exercises 248

 6.2 The Theorem of Pólya–Redfield 249

 Exercises 260

 6.3 Cycle Index 262

 Exercises 269

 6.4 Symmetries on N and R 270

 Exercises 276

 Highlight: Patterns of Polyominoes 278

 Notes and References 285

Part III: Topics

7 The Catalan Connection 289

 7.1 Catalan Matrices and Orthogonal Polynomials 290

 Exercises 297

 7.2 Catalan Numbers and Lattice Paths 300

 Exercises 305

 7.3 Generating Functions and Operator Calculus 306

 Exercises 320

 7.4 Combinatorial Interpretation of Catalan Numbers ... 323

 Exercises 333

 Highlight: Chord Diagrams 337

 Notes and References 344

8 Symmetric Functions 345

 8.1 Symmetric Polynomials and Functions 345

 Exercises 349

 8.2 Homogeneous Symmetric Functions 350

 Exercises 355

 8.3 Schur Functions 356

 Exercises 366

 8.4 The RSK Algorithm 367

 Exercises 378

 8.5 Standard Tableaux 380

 Exercises 383

 Highlight: Hook-Length Formulas 385

 Notes and References 391

9	Counting Polynomials	393
9.1	The Tutte Polynomial of Graphs	393
	Exercises	405
9.2	Eulerian Cycles and the Interlace Polynomial	407
	Exercises	419
9.3	Plane Graphs and Transition Polynomials	420
	Exercises	432
9.4	Knot Polynomials	434
	Exercises	443
	Highlight: The BEST Theorem	445
	Notes and References	449
10	Models from Statistical Physics	451
10.1	The Dimer Problem and Perfect Matchings	451
	Exercises	465
10.2	The Ising Problem and Eulerian Subgraphs	467
	Exercises	480
10.3	Hard Models	481
	Exercises	489
10.4	Square Ice	490
	Exercises	504
	Highlight: The Rogers–Ramanujan Identities	506
	Notes and References	517
	Solutions to Selected Exercises	519
	Chapter 1	519
	Chapter 2	521
	Chapter 3	524
	Chapter 4	528
	Chapter 5	529
	Chapter 6	533
	Chapter 7	536
	Chapter 8	540
	Chapter 9	544
	Chapter 10	547
	Notation	553
	Index	557