

Table of Contents

Part I Point Estimation and Linear Regression

1	Fundamentals	3
1.1	Linear Models	3
1.1.1	Application of Linear Models	3
1.1.2	Types of Linear Models	6
1.1.3	Proceeding with Linear Models	7
1.1.4	A Preliminary Example	8
1.2	Decision Theory and Point Estimation	12
1.2.1	Decision Rule	12
1.2.2	Non-operational Decision Rule	14
1.2.3	Loss and Risk	15
1.2.4	Choosing a Decision Rule	16
1.2.5	Admissibility	18
1.2.6	Squared Error Loss	20
1.2.7	Matrix Valued Squared Error Loss	22
1.2.8	Alternative Loss Functions	26
1.3	Problems	29
2	The Linear Regression Model	33
2.1	Assumptions	33
2.2	Ordinary Least Squares Estimation	36
2.2.1	The Principle of Least Squares	37
2.2.2	Coefficient of Determination R^2	39
2.2.3	Predictive Loss	41
2.2.4	Least Squares Variance Estimator	44
2.2.5	Properties of the Ordinary Least Squares Estimator	45
2.2.6	Properties Under Normality	46
2.3	Optimality of Least Squares Estimation	47
2.3.1	Linear Unbiased Estimation	48
2.3.2	Gauss-Markov Theorem	49
2.3.3	Normality Assumption	51
2.3.4	Admissibility	52
2.4	Unreliability of Least Squares Estimation	53
2.4.1	Estimation of the Covariance Matrix	53

VIII Table of Contents

2.4.2	Unbiased Versus Biased Estimation	55
2.4.3	Collinearity	57
2.4.4	Consistency	65
2.4.5	Biased Estimation	67
2.5	Inadmissibility of the Ordinary Least Squares Estimator	69
2.5.1	The Reparameterized Regression Model	69
2.5.2	Risk Comparison of Least Squares and Stein Estimator	71
2.5.3	An Example for Stein Estimation	77
2.5.4	Admissibility	78
2.6	Problems	80

Part II Alternatives to Least Squares Estimation

3	Alternative Estimators	89
3.1	Restricted Least Squares Estimation	89
3.1.1	The Principle of Restricted Least Squares	90
3.1.2	The Parameter Space	91
3.1.3	Properties of Restricted Least Squares Estimator	92
3.1.4	Risk Comparison of Restricted and Ordinary Least Squares Estimator	94
3.1.5	Pretest Estimation	98
3.2	Other Types of Restriction	102
3.2.1	Stochastic Linear Restrictions	102
3.2.2	Inequality Restrictions	103
3.2.3	Elliptical Restrictions	105
3.3	Principal Components Estimator	106
3.3.1	Preliminary Considerations	107
3.3.2	Properties of the Principal Components Estimator	108
3.3.3	Drawbacks of the Principal Components Estimator	113
3.3.4	The Marquardt Estimator	114
3.4	Ridge Estimator	115
3.4.1	Preliminary Considerations	115
3.4.2	Properties of the Linear Ridge Estimator	118
3.4.3	The Choice of the Ridge Parameter	123
3.4.4	Standardization	128
3.4.5	Ridge and Restricted Least Squares Estimator	134
3.4.6	Ridge and Principal Components Estimator	138
3.4.7	Jackknife Modified Ridge Estimator	141
3.4.8	Iteration Estimator	144
3.4.9	An Example for Ridge Estimation	147
3.5	Shrinkage Estimator	150
3.5.1	Preliminary Considerations	150
3.5.2	Risk Comparison to Ordinary Least Squares	152
3.5.3	The Choice of the Shrinkage Parameter	153

3.5.4	Direction Modified Shrinkage Estimators	157
3.6	General Ridge Estimator	163
3.6.1	A Class of Estimators	163
3.6.2	Risk Comparison of General Ridge and Ordinary Least Squares Estimator	164
3.7	Linear Minimax Estimator	165
3.7.1	Preliminary Considerations	166
3.7.2	Inequality Restrictions	167
3.7.3	Linear Minimax Solutions	169
3.7.4	Alternative Approaches	174
3.7.5	Admissibility	179
3.8	Linear Bayes Estimator	181
3.8.1	Preliminary Considerations	181
3.8.2	Characterization of Linear Bayes Estimators	182
3.8.3	Non-Operational Bayes Solutions	184
3.8.4	A-priori Assumptions	185
3.9	Robust Estimator	188
3.9.1	Preliminary Considerations	189
3.9.2	Weighted Least Squares Estimation	192
3.9.3	The l_1 Estimator	197
3.9.4	M Estimator	202
3.9.5	Robust Ridge Estimator	203
3.10	Problems	204
4	Linear Admissibility	213
4.1	Preliminary Considerations	213
4.2	Linear Admissibility in the Non-Restricted Model	215
4.2.1	Linear Admissibility in the Simple Mean Shift Model .	215
4.2.2	Characterization of Linearly Admissible Estimators .	219
4.2.3	Ordinary Least Squares and Linearly Admissible Estimator	222
4.2.4	Linear Transforms of Ordinary Least Squares Estimator	224
4.2.5	Linear Admissibility of Known Estimators	226
4.2.6	Shrinkage Property and Linear Admissibility	229
4.2.7	Convex Combination of Estimators	231
4.2.8	Linear Bayes Estimator	236
4.3	Linear Admissibility Under Linear Restrictions	238
4.3.1	The Assumption of a Full Rank Restriction Matrix .	238
4.3.2	Restricted Estimator	239
4.3.3	Characterization of Linearly Admissible Estimators .	244
4.4	Linear Admissibility Under Elliptical Restrictions	246
4.4.1	Characterization of Linearly Admissible Estimators .	247
4.4.2	Linear Admissibility of Certain Linear Estimators .	249
4.4.3	Admissible Improvements Over Ordinary Least Squares	251
4.5	Problems	254

Part III Miscellaneous Topics

5 The Covariance Matrix of the Error Vector	259
5.1 Estimation of the Error Variance	259
5.1.1 The Sample Variance	259
5.1.2 Nonnegative Unbiased Estimation	260
5.1.3 Optimality of the Least Squares Variance Estimator	261
5.1.4 Non-Admissibility of the Least Squares Variance Estimator	262
5.2 Non-Scalar Covariance Matrix	265
5.2.1 Preliminary Considerations	266
5.2.2 The Transformed Model	267
5.2.3 Two-Stage Estimation	270
5.3 Occurrence of Non-Scalar Covariance Matrices	271
5.3.1 Seemingly Unrelated Regression	271
5.3.2 Heteroscedastic Errors	274
5.3.3 Equicorrelated Errors	280
5.3.4 Autocorrelated Errors	281
5.4 Singular Covariance Matrices	284
5.5 Equality of Ordinary and Generalized Least Squares	287
5.6 Problems	289
6 Regression Diagnostics	293
6.1 Selecting Independent Variables	293
6.1.1 Mallows' C_p	293
6.1.2 Stepwise Regression	294
6.1.3 Alternative Criteria	295
6.2 Assessing Goodness of Fit	298
6.3 Diagnosing Collinearity	302
6.3.1 Variance Inflation Factors	302
6.3.2 Scaled Condition Indexes	305
6.4 Inspecting Residuals	308
6.4.1 Normal Quantile Plot	310
6.4.2 Residuals Versus Fitted Values Plot	310
6.4.3 Further Residual Plots	312
6.5 Finding Influential Observations	313
6.5.1 Leverage	313
6.5.2 Influential Observations	314
6.5.3 Collinearity-Influential Observations	317
6.6 Testing Model Assumptions	317
6.6.1 Preliminary Considerations	317
6.6.2 Testing for Heteroscedasticity	318
6.6.3 Testing for Autocorrelation	320
6.6.4 Testing for Non-Normality	322

6.6.5	Testing for Non-Linearity	323
6.6.6	Testing for Outlier	326
6.7	Problems	327
A	Matrix Algebra	331
A.1	Preliminaries	331
A.1.1	Matrices and Vectors	331
A.1.2	Elementary Operations	332
A.1.3	Rank of a Matrix	333
A.1.4	Subspaces and Matrices	334
A.1.5	Partitioned Matrices	335
A.1.6	Kronecker Product	336
A.1.7	Moore-Penrose Inverse	337
A.2	Common Pitfalls	338
A.3	Square Matrices	339
A.3.1	Specific Square Matrices	339
A.3.2	Trace and Determinant	344
A.3.3	Eigenvalue and Eigenvector	346
A.3.4	Vector and Matrix Norm	350
A.3.5	Definiteness	351
A.4	Symmetric Matrix	351
A.4.1	Eigenvalues	351
A.4.2	Spectral Decomposition	352
A.4.3	Rayleigh Ratio	353
A.4.4	Definiteness	353
A.5	Löwner Partial Ordering	356
B	Stochastic Vectors	359
B.1	Expectation and Covariance	359
B.2	Multivariate Normal Distribution	362
B.3	χ^2 Distribution	365
B.4	F Distribution	367
C	An Example Analysis with R	369
C.1	Problem and Goal	369
C.2	The Data	370
C.3	The Choice of Variables	371
C.3.1	The Full Model	371
C.3.2	Stepwise Regression	373
C.3.3	Collinearity Diagnostics	374
C.4	Further Diagnostics	375
C.4.1	Residuals	375
C.4.2	Influential Observations	377
C.5	Prediction	378

XII Table of Contents

References	381
Index	389