Contents

1 Introduction 1

2 Variational problems with linear growth: the general setting 13
2.1 Construction of a solution for the dual problem which is of class $W^1_{2,loc}(\Omega;\mathbb{R}^{nN})$ 14
2.1.1 The dual problem 14
2.1.2 Regularization 16
2.1.3 $W^1_{2,loc}$-regularity for the dual problem 19
2.2 A uniqueness theorem for the dual problem 20
2.3 Partial $C^{1,\alpha}$- and $C^{0,\alpha}$-regularity, respectively, for generalized minimizers and for the dual solution 25
2.3.1 Partial $C^{1,\alpha}$-regularity of generalized minimizers 26
2.3.2 Partial $C^{0,\alpha}$-regularity of the dual solution 29
2.4 Degenerate variational problems with linear growth 32
2.4.1 The duality relation for degenerate problems 33
2.4.2 Application: an intrinsic regularity theory for σ 39

3 Variational integrands with (s,μ,q)-growth 41
3.1 Existence in Orlicz-Sobolev spaces 42
3.2 The notion of (s,μ,q)-growth – examples 44
3.3 A priori gradient bounds and local $C^{1,\alpha}$-estimates for scalar and structured vector-valued problems 50
3.3.1 Regularization 52
3.3.2 A priori L^q-estimates 54
3.3.3 Proof of Theorem 3.16 61
3.3.4 Conclusion 67
3.4 Partial regularity in the general vectorial setting 69
3.4.1 Regularization 69
3.4.2 A Caccioppoli-type inequality 70
3.4.3 Blow-up 72
3.4.3.1 Blow-up and limit equation 74
3.4.3.2 An auxiliary proposition 76
3.4.3.3 Strong convergence 83
3.4.3.4 Conclusion 86
3.4.4 Iteration 87

3.5 Comparison with some known results 89
3.5.1 The scalar case 89
3.5.2 The vectorial setting 90
3.6 Two-dimensional anisotropic variational problems 91

4 Variational problems with linear growth: the case of μ-elliptic integrands 97
4.1 The case $\mu < 1 + 2/n$ 100
4.1.1 Regularization 101
4.1.2 Some remarks on the dual problem 101
4.1.3 Proof of Theorem 4.4 103
4.2 Bounded generalized solutions 104
4.2.1 Regularization 108
4.2.2 The limit case $\mu = 3$ 111
4.2.2.1 Higher local integrability 111
4.2.2.2 The independent variable 113
4.2.3 L^p-estimates in the case $\mu < 3$ 116
4.2.4 A priori gradient bounds 118
4.3 Two-dimensional problems 122
4.3.1 Higher local integrability in the limit case 123
4.3.2 The case $\mu < 3$ 129
4.4 A counterexample 132

5 Bounded solutions for convex variational problems with a wide range of anisotropy 141
5.1 Vector-valued problems 142
5.2 Scalar obstacle problems 149

6 Anisotropic linear/superlinear growth in the scalar case . . . 161

A Some remarks on relaxation 173
A.1 The approach known from the minimal surface case 174
A.2 The approach known from the theory of perfect plasticity 176
A.3 Two uniqueness results 181

B Some density results 185
B.1 Approximations in BV 185
B.2 A density result for $\mathcal{U} \cap L(c)$ 191
B.3 Local comparison functions 194

C Brief comments on steady states of generalized Newtonian fluids 199

D Notation and conventions 205

References 207

Index 215