

# Contents

<b>1. Introduction</b> .....	1
1.1 Motivation .....	1
1.2 Issues .....	4
1.3 Scope of the Book .....	5
1.4 Organization of the Book .....	5
<b>2. Overview of Spatial Reasoning and Planning Techniques</b> ..	9
2.1 Computer-Aided Kinematic Design of Mechanisms .....	9
2.2 Geometric Path Planning .....	11
2.2.1 Path Search in Configuration Space .....	11
2.2.2 Path Finding Based on Direct Free-Space Characterization .....	13
2.2.3 Local Path Planning .....	14
2.3 Qualitative Reasoning .....	15
2.3.1 Qualitative Mechanism Analysis .....	15
2.3.2 Qualitative Spatial Reasoning .....	16
2.3.3 Qualitative Robotics .....	18
2.3.4 Qualitative Physics .....	18
2.4 Simulated Annealing .....	20
<b>3. Interesting Problems in Spatial Reasoning and Planning</b> ..	23
3.1 Terminology and Notation .....	23
3.2 The Problems .....	26
3.3 Assumptions .....	29
<b>4. How to Represent Qualitative Spatial Relationships</b> .....	31
4.1 Qualitative Distance .....	33
4.2 Qualitative Angle .....	34
4.3 Notes on <i>Label-based Distance and Angle Descriptions</i> .....	36
4.4 Completeness .....	37

4.5	Minimum-Spanning Edge ( $m$ -Edge) between Two Polygons . . .	37
4.6	Qualitative Location in a Convex Polygonal Environment . . . .	38
4.6.1	Qualitative Location . . . . .	41
4.7	Graphic Representation of the $m$ -Edge Partitioned Free-Space	42
4.8	Notes on <i>Qualitative Location</i> . . . . .	42
<b>5.</b>	<b>Methodology of Spatial Reasoning and Planning</b> . . . . .	<b>47</b>
5.1	Spatial Inferencing . . . . .	48
5.1.1	Qualitative Trigonometry ( $QT$ ) . . . . .	48
5.1.2	Qualitative Arithmetic ( $QA$ ) and Propagation . . . . .	50
5.1.3	Inferencing . . . . .	50
5.2	Envisionments . . . . .	54
5.3	Spatial Planning in $Q$ -Space . . . . .	55
5.3.1	Qualitative Route . . . . .	56
5.3.2	Clearance Measurements of a Qualitative Route . . . . .	56
5.4	Quantitative Configuration Generation with Simulated Annealing . . . . .	60
<b>6.</b>	<b>How to Reason about Mechanism Configurations</b> . . . . .	<b>63</b>
6.1	An Overview of the Method . . . . .	63
6.2	Qualitative Configuration Analysis . . . . .	65
6.2.1	Examples . . . . .	67
6.3	Quantitative Configuration Generation . . . . .	71
6.3.1	Examples . . . . .	72
6.4	Discussions . . . . .	87
6.4.1	Features and Advantages . . . . .	87
6.4.2	Limitations . . . . .	88
6.5	Kinematic State Transitions in CSV Mechanisms . . . . .	89
6.5.1	Vertex-Contact Configurations of CSV Mechanisms . . .	91
6.5.2	Placement of Vertices in VC Configurations . . . . .	91
6.5.3	Identification of Kinematic State Transitions . . . . .	93
6.6	Summary . . . . .	100
<b>7.</b>	<b>How to Reason about Velocity Relationships</b> . . . . .	<b>101</b>
7.1	Instantaneous Rotation Center . . . . .	101
7.2	Velocity Relationship Analysis . . . . .	103
7.3	Examples . . . . .	109
7.4	Notes on the Application of <i>Velocity Analysis</i> . . . . .	112
7.5	Relative Motion Method of Analyzing Velocities . . . . .	112

7.5.1	Axioms and Theorems in Revolute or Prismatic- Pairing Body Motion .....	113
7.5.2	Kinematic Modeling .....	115
7.6	Qualitative Analysis of Relative Velocities .....	115
7.6.1	Solving Velocity Constraint Equations .....	115
7.6.2	An Algorithm for Determining Linear Velocities .....	116
7.7	An Example .....	118
7.8	Summary .....	121
<b>8.</b>	<b>How to Plan Robot Motions .....</b>	<b>123</b>
8.1	An Overview of the Method .....	123
8.2	Qualitative Route Planning in the $m$ -Edge Partitioned Euclidean Free-Space .....	124
8.2.1	Eliminating Dead-End Regions .....	125
8.2.2	Representing Path-Segment Invariants .....	126
8.2.3	An Algorithm for Finding Qualitative Routes .....	127
8.3	Constructing Exact Paths from Qualitative Routes .....	129
8.3.1	The Composition of an Exact Path .....	129
8.3.2	Randomized Search for Exact Path Segments .....	130
8.3.3	An Algorithm for Computing Exact Path Segments ...	130
8.4	Graphical Simulations .....	131
8.4.1	Examples .....	132
8.5	Discussions .....	144
8.5.1	Efficiency .....	144
8.5.2	Near-Obstacle Paths .....	145
8.5.3	Comparison with Other Free-Space-Based Approaches	145
8.5.4	Comparison with Other Monte-Carlo Path-Planning Approaches .....	146
8.5.5	Limitations .....	147
<b>9.</b>	<b>How to Make Spatial Measurements and Maps .....</b>	<b>149</b>
9.1	Mapping .....	150
9.2	$m$ -Uncertainty and FS Theory .....	151
9.3	Incorporating $m$ -Uncertainty .....	153
9.4	Collective Spatial Map Construction .....	154
9.4.1	Related Work on Spatial Map Construction .....	155
9.4.2	The Problem .....	156
9.5	Self-Organization of a Potential Map .....	158
9.5.1	Coordinate Systems for an Agent .....	158

9.5.2	Proximity Measurements .....	159
9.5.3	Distance Association within a Neighboring Region .....	159
9.5.4	Incremental Self-Organization of a Potential Map .....	160
9.6	Experiments .....	162
9.6.1	Experimental Design .....	163
9.6.2	Comparison with a Non-Adaptive Mode .....	164
9.6.3	Experimental Results and Comparisons .....	165
9.7	Summary .....	168
<b>10.</b>	<b>Concluding Remarks .....</b>	<b>171</b>
10.1	Key Concepts Revisited .....	171
10.2	Practical Application .....	172
10.2.1	Computer-Aided Mechanism Analysis .....	172
10.2.2	Robot Compliant Task Analysis .....	173
10.2.3	Robot Path Planning .....	174
10.3	Limitations .....	174
10.4	Future Challenges .....	175
10.4.1	Simulated Annealing .....	175
10.4.2	Local Path Planning near $m$ -Edges .....	175
10.4.3	Other Heuristic Search Strategies for Qualitative Route Planning .....	176
10.4.4	Incorporating a Continuous Manipulator Model .....	176
10.4.5	Extensions to Complex Mechanisms and Non-Convex Obstacles .....	176
	<b>Appendices .....</b>	<b>179</b>
	<b>A. Grid-Based Representation for Totally Ordered <math>\mathcal{Q}</math>-space ..</b>	<b>179</b>
	<b>B. The Boltzmann Distribution in Simulated Annealing .....</b>	<b>183</b>
	<b>C. Qualitative Route Search Based on <math>A^*</math> Algorithm .....</b>	<b>185</b>
	<b>Bibliography .....</b>	<b>187</b>
	<b>Index .....</b>	<b>198</b>