

Contents

1	Introduction	1
1.1	Motivations and Purpose of the Book	1
1.2	Structures in Statistical Physics: A New Perspective	2
1.3	Structures in Statistical Physics: The Methods	8
1.4	Applications to Cosmology	11
1.5	Perspectives for the Future	22

Part I Statistical Methods

2	Uniform and Correlated Mass Density Fields	27
2.1	Introduction	27
2.2	Basic Statistical Properties and Concepts	31
2.2.1	Spatial Averages and Ergodicity	34
2.2.2	Homogeneity and Homogeneity Scale	34
2.3	Correlation Functions	35
2.3.1	Characteristic Function and Cumulants Expansion	36
2.3.2	Correlation Length	39
2.3.3	Other Properties of the Reduced Two-Point Correlation Function	40
2.3.4	Mass Variance	41
2.4	Poisson Point Process	44
2.5	Stochastic Point Processes with Spatial Correlations	46
2.5.1	Conditional Properties	48
2.5.2	Integrated Conditional Properties	50
2.5.3	Detection of the Homogeneity Scale of a Discrete SPP	50
2.6	Nearest Neighbor Probability Density in Point Processes	52
2.6.1	Poisson Case	52
2.6.2	Particle Distributions with Spatial Correlations	54
2.7	Gaussian Continuous Stochastic Fields	55
2.8	Power-Laws and Self-Similarity	58
2.9	Mass Function and Probability Distribution	61
2.10	The Random Walk and the Central Limit Theorem	64

2.10.1	Probability Distribution of Mass Fluctuations in Large Volumes	68
2.11	Gaussian Distribution as the Most Probable Probability Distribution	69
2.12	Summary and Discussion	71
3	The Power Spectrum and the Classification of Stationary Stochastic Fields	73
3.1	Introduction	73
3.2	General Properties	73
3.2.1	Mathematical Definitions	73
3.2.2	Limit Conditions	76
3.3	The Power Spectrum for the Poisson Point Process and Other SPP	77
3.4	The Power Spectrum and the Mass Variance: A Complete Classification	78
3.4.1	The Complete Classification of Mass Fluctuations versus Power Spectrum	83
3.5	Super-Homogeneous Mass Density Fields	84
3.5.1	The Lattice Particle Distribution	85
3.5.2	The One Component Plasma	88
3.6	Further Analysis of Gaussian Fields	91
3.6.1	Real Space Composition of Gaussian Fields, Correlation Length and Size of Structures	95
3.7	Summary and Discussion	96
4	Fractals	101
4.1	Introduction	101
4.2	The Metric Dimension	102
4.3	Conditional Density	107
4.3.1	Conditional Density and Smooth Radial Particle Distributions	109
4.3.2	Statistically Homogeneous and Isotropic Distribution of Radial Density Profiles	113
4.3.3	Nearest Neighbor Probability Density for Radial and Fractal Point-Particle Distributions	113
4.4	The Two-Point Conditional Density	116
4.5	The Conditional Variance in Spheres	118
4.6	Corrections to Scaling	119
4.6.1	Correction to Scaling: Deterministic Fractals	120
4.6.2	Correction to Scaling: Random Fractals	124
4.7	Fractal with a Crossover to Homogeneity	127
4.8	Correlation, Fractals and Clustering	127
4.9	Probability Distribution of Mass Fluctuations in a Fractal	130

4.10	Intersection of Fractals	132
4.11	Morphology and Voids	134
4.12	Angular and Orthogonal Projection of Fractal Sets	134
4.12.1	On the Uniformity of the Angular Projection	137
4.13	Summary and Discussion	141
5	Multifractals and Mass Distributions	143
5.1	Introduction	143
5.2	Basic Definitions	144
5.3	Deterministic Multifractals	145
5.4	The Multifractal Spectrum	149
5.5	Random Multifractals	151
5.6	Self-Similarity of Fluctuations and Multifractality in Temporal Multiplicative Processes	154
5.7	Spatial Correlation in Multifractals	158
5.8	Multifractals and “Mass” Distributions	159
5.9	Summary and Discussion	161

Part II Applications to Cosmology

6	Fluctuations in Standard Cosmological Models:	
	A Real Space View	167
6.1	Introduction	167
6.2	Basic Properties of Cosmological Density Fields	167
6.3	The Cosmological Origin of the HZ Spectrum	171
6.4	The Real Space Correlation Function of CDM/HDM Models	173
6.5	$P(0) = 0$ and Constraints in a Finite Sample	177
6.6	CMBR Anisotropies in Direct Space	179
6.6.1	CMBR Anisotropies and the Matter Power Spectrum ..	180
6.6.2	The Origin of Oscillations in the Power Spectrum	183
6.6.3	A Simple Example of k -Oscillations	184
6.6.4	Oscillations in the CDM PS	185
6.6.5	Oscillations in the CMBR Anisotropies	187
6.7	Summary and Discussion	189
7	Discrete Representation of Fluctuations in Cosmological Models	193
7.1	Introduction	193
7.2	Discrete versus Continuous Density Fields	194
7.3	Super-Homogeneous Systems in Statistical Physics	196
7.4	HZ as Equilibrium of a Modified OCP	197
7.5	A First Approximation to the Effect of Displacement Fields ..	199
7.6	Displacement Fields: Formulation of the Problem	200

7.7	Effects of Displacements on One and Two-Point Properties of the Particle Distribution	203
7.7.1	Uncorrelated Displacements	206
7.7.2	Asymptotic Behavior of $P(\mathbf{k})$ for Small k	208
7.7.3	The Shuffled Lattice with Uncorrelated Displacements	209
7.8	Correlated Displacements	212
7.8.1	Correlated Gaussian Displacement Field	214
7.9	Summary and Discussion	217
8	Galaxy Surveys: An Introduction to Their Analysis	219
8.1	Introduction	219
8.2	Basic Assumptions and Definitions	220
8.3	Galaxy Catalogs and Redshift	221
8.4	Volume Limited Samples	224
8.5	The Discovery of Large Scale Structure in Galaxy Catalogs	227
8.6	Standard Characterization of Galaxy Correlations and the Assumption of Homogeneity	228
8.7	Summary and Discussion	233
9	Characterizing the Observed Distribution of Visible Matter I: The Conditional Average Density in Galaxy Catalogs	235
9.1	Introduction	235
9.2	The Conditional Average Density in Finite Samples	236
9.3	Sample Size Smaller than the Homogeneity Scale	240
9.3.1	The Reduced Correlation Function for a Particle Distribution with Fractal Behavior in the Sample	240
9.4	Sample Size Greater Than the Homogeneity Scale	242
9.4.1	Critical Case	243
9.4.2	Substantially Poisson Case	244
9.4.3	Super-Homogeneous Case	245
9.4.4	Some Remarks	245
9.5	Estimating the Average Conditional Density in a Finite Sample	246
9.5.1	Estimators of the Average Conditional Density	247
9.5.2	Effective Depth of Samples	250
9.6	The Average Conditional Density (FS) in Real Galaxy Catalogs	250
9.6.1	Normalization of the Average Conditional in Different VL Samples	257
9.6.2	Estimation of the Conditional Average Luminosity Density	259
9.6.3	Measuring the Average Mass Density Ω from Redshift Surveys	260
9.7	Summary and Discussion	263

10	Characterizing the Observed Distribution of Visible Matter II: Number Counts and Their Fluctuations	265
10.1	Introduction	265
10.2	Number Counts in Real Space	266
10.3	Number Counts as a Function of Apparent Magnitude	268
10.3.1	Poisson Distribution	268
10.3.2	Simple Fractal Distribution	271
10.3.3	Effect of Long-Ranged Correlations in Homogeneous Distributions	273
10.4	Normalization of the Magnitude Counts to Real Space Properties in Euclidean Space	276
10.4.1	Average Distance	276
10.4.2	Normalization of Distance to Magnitude Counts	277
10.5	Galaxy Counts in Real Catalogs	278
10.5.1	Real Space Counts	279
10.5.2	Magnitude Space Counts	283
10.6	Summary and Discussion	288
11	Luminosity in Galaxy Correlations	291
11.1	Introduction	291
11.2	Standard Methods for the Estimation of the Luminosity Function	292
11.3	Multifractality, Luminosity and Space Distributions	293
11.4	Summary and Discussion	297
12	The Distribution of Galaxy Clusters	299
12.1	Introduction	299
12.2	Cluster Correlations and Multifractality	300
12.3	Galaxy Cluster Correlations	303
12.3.1	The Average Conditional Density for Galaxy Clusters	306
12.3.2	Galaxy-Cluster Mismatch	306
12.4	Luminosity Bias and the Richness-Clustering Relation	308
12.5	Summary and Discussion	311
13	Biasing a Gaussian Random Field and the Problem of Galaxy Correlations	313
13.1	Introduction	313
13.2	Biasing of Gaussian Random Fields	314
13.3	Biasing and Real Space Correlation Properties	318
13.4	Biasing and the Power Spectrum	325
13.5	Summary and Discussion	330

14 The Gravitational Field in Stochastic Particle Distributions 335

14.1 Introduction 335

14.2 Nearest Neighbor Force Distribution 336

14.3 Gravitational Force PDF in a Poisson Particle Distribution 338

14.4 Gravitational Force in Weakly Correlated Particle Distributions: the Gauss-Poisson Case 342

14.5 Generalization of the Holtzmark Distribution to the Gauss-Poisson Case 343

14.5.1 Large F Expansion 344

14.5.2 Small F Expansion 347

14.5.3 Comparison with Simulations 347

14.5.4 Nearest-Neighbor Approximation for the Gauss-Poisson Case 348

14.6 Gravitational Force in Fractal Point Distributions 350

14.7 An Upper Limit in the Fractal Case 351

14.8 Average Quadratic Force in a Fractal 354

14.9 The General Importance of the Force-Force Correlation 358

14.10 Summary and Discussion 360

Part III Appendixes

A Scaling Behavior of the Characteristic Function for Asymptotically Small Values of k 365

B Fractal Algorithms 369

B.1 Cantor Set and Random Cantor Set 369

B.2 Levy Flight 372

B.3 Random Trema Dust 372

C Cosmological Models: Basic Relations 375

C.1 Cosmological Parameters 376

C.1.1 Comoving (Radial) Distance 376

C.1.2 Comoving (Transverse) Distance 377

C.1.3 Luminosity Distance 377

C.1.4 Magnitude 377

C.2 Cosmological Corrections in the Analysis of Redshift Surveys 378

C.2.1 Flat Cosmologies: FMD and FLD 378

C.2.2 Open Model: OBD 380

D	Cosmological and k-Corrections to Number Counts	381
	D.1 k-Corrections	381
	D.2 k-Corrections and the Radial Number Counts	382
	D.3 Dependence on the Cosmological Model	383
E	Fractal Matter in an Open FRW Universe	385
	E.1 Introduction	385
	E.2 Friedmann Solution in an Empty Universe	386
	E.3 Curvature Dominated Phase	387
	E.4 Radiation Dominated Era	390
	E.5 Fluctuations in the CMBR	391
	E.6 Other Remarks	392
F	Errors in Full Shell Estimators	395
	F.1 Bias and Variance of Estimators	395
	F.2 Unconditional Average Density	396
	F.3 Conditional Number of Points in a Sphere	397
	F.4 Integrated Conditional Density	398
	F.5 Conditional Average Density in Shells	399
	F.6 Reduced Two-Point Correlation Function	402
G	Non Full-Shell Estimation of Two Point Correlation Properties	405
	G.1 Estimators with Simple Weightings	406
	G.2 Other Pair Counting Estimators	407
	G.3 Estimation of the Conditional Density Beyond R_s	409
H	Estimation of the Power Spectrum	411
	References	413
	Index	421