

# Table of Contents

<b>1</b>	<b>Topology optimization by distribution of isotropic material</b>	<b>1</b>
1.1	Problem formulation and parametrization of design	1
1.1.1	Minimum compliance design	2
1.1.2	Design parametrization	4
1.1.3	Alternative problem forms	8
1.2	Solution methods	9
1.2.1	Conditions of optimality	9
1.2.2	Implementation of the optimality criteria method	12
1.2.3	Sensitivity analysis and mathematical programming methods	15
1.2.4	Implementation - the general concept	21
1.2.5	Topology optimization as a design tool	24
1.3	Complications	28
1.3.1	Mesh-refinement and existence of solutions	28
1.3.2	The checkerboard problem	39
1.3.3	Non-uniqueness, local minima and dependence on data	46
1.4	Combining topology and shape design	47
1.5	Variations of the theme	53
1.5.1	Multiple loads	53
1.5.2	Variable thickness sheets	54
1.5.3	Plate design	58
1.5.4	Other interpolation schemes with isotropic materials	60
1.5.5	Design parametrization with wavelets	66
1.5.6	Alternative approaches	68
<b>2</b>	<b>Extensions and applications</b>	<b>71</b>
2.1	Problems in dynamics	72
2.1.1	Free vibrations and eigenvalue problems	72
2.1.2	Forced vibrations	76
2.2	Buckling problems	77
2.3	Stress constraints	79
2.3.1	A stress criterion for the SIMP model	80
2.3.2	Solution aspects	81
2.4	Pressure loads	84
2.5	Geometrically non-linear problems	86

2.5.1	Problem formulation and objective functions . . . . .	86
2.5.2	Choice of objective function for stiffness optimization . . . . .	87
2.5.3	Numerical problems and ways to resolve them . . . . .	89
2.5.4	Examples . . . . .	90
2.6	Synthesis of compliant mechanisms . . . . .	94
2.6.1	Problem setting . . . . .	95
2.6.2	Output control . . . . .	97
2.6.3	Path generating mechanisms . . . . .	98
2.6.4	Linear modelling . . . . .	100
2.6.5	Linear vs. non-linear modelling . . . . .	101
2.6.6	Design of thermal actuators . . . . .	104
2.6.7	Computational issues . . . . .	104
2.7	Design of supports . . . . .	108
2.8	Alternative physics problems . . . . .	110
2.8.1	Multiphysics problems . . . . .	111
2.8.2	MicroElectroMechanical Systems (MEMS) . . . . .	113
2.8.3	Stokes flow problems . . . . .	115
2.9	Optimal distribution of multiple material phases . . . . .	117
2.9.1	One material structures . . . . .	118
2.9.2	Two material structures without void . . . . .	119
2.9.3	Two material structures with void . . . . .	120
2.9.4	Examples of multiphase design . . . . .	121
2.10	Material design . . . . .	122
2.10.1	Numerical homogenization and sensitivity analysis . . . . .	123
2.10.2	Objective functions for material design . . . . .	124
2.10.3	Material design results . . . . .	126
2.11	Wave propagation problems . . . . .	138
2.11.1	Modelling of wave propagation . . . . .	140
2.11.2	Optimization of band gap materials . . . . .	144
2.11.3	Optimization of band gap structures . . . . .	146
2.12	Various other applications . . . . .	148
2.12.1	Material design for maximum buckling load . . . . .	148
2.12.2	Crashworthiness . . . . .	149
2.12.3	Bio-mechanical simulations . . . . .	151
2.12.4	Applications in the automotive industry . . . . .	152
<b>3</b>	<b>Design with anisotropic materials . . . . .</b>	<b>159</b>
3.1	The homogenization approach . . . . .	160
3.1.1	Parametrization of design . . . . .	160
3.1.2	The homogenization formulas . . . . .	162
3.1.3	Implementation of the homogenization approach . . . . .	167
3.1.4	Conditions of optimality for compliance optimization - rotations and densities . . . . .	169
3.2	Optimized energy functionals . . . . .	173

3.2.1	Combining local optimization of material properties and spatial optimization of material distribution . . . . .	174
3.2.2	A hierarchical solution procedure . . . . .	176
3.3	Optimized energy functionals for the homogenization modelling . . . . .	179
3.3.1	The stress based analysis of optimal layered materials .	179
3.3.2	The strain based problem of optimal layered materials	182
3.3.3	The limiting case of Michell's structural continua . . . . .	183
3.3.4	Comparing optimal energies . . . . .	186
3.3.5	Optimal energies and the checkerboard problem . . . . .	189
3.4	Design with a free parametrization of material . . . . .	190
3.4.1	Problem formulation for a free parametrization of design . . . . .	191
3.4.2	The solution to the optimum local anisotropy problems	192
3.4.3	Analysis of the reduced problems . . . . .	196
3.4.4	Numerical implementation and examples . . . . .	200
3.4.5	Free material design and composite structures . . . . .	202
3.5	Plate design with composite materials . . . . .	204
3.5.1	The homogenization approach for Kirchhoff plates . . . . .	204
3.5.2	Minimum compliance design of laminated plates . . . . .	206
3.6	Optimal topology design with a damage related criterion . . . . .	214
3.6.1	A damage model of maximizing compliance . . . . .	215
3.6.2	Design problems . . . . .	218
<b>4</b>	<b>Topology design of truss structures . . . . .</b>	<b>221</b>
4.1	Problem formulation for minimum compliance truss design . . . . .	223
4.1.1	The basic problem statements in displacements . . . . .	223
4.1.2	The basic problem statements in member forces . . . . .	226
4.1.3	Problem statements including self-weight and reinforcement . . . . .	229
4.2	Problem equivalence and globally optimized energy functionals	230
4.2.1	Conditions of optimality . . . . .	230
4.2.2	Reduction to problem statements in bar volumes only .	233
4.2.3	Reduction to problem statements in displacements only	235
4.2.4	Linear programming problems for single load problems	238
4.2.5	Reduction to problem statements in stresses only . . . . .	240
4.2.6	Extension to contact problems . . . . .	242
4.3	Computational procedures and examples . . . . .	245
4.3.1	An optimality criteria method . . . . .	246
4.3.2	A non-smooth descent method . . . . .	247
4.3.3	SDP and interior point methods . . . . .	248
4.3.4	Examples . . . . .	250
4.4	Extensions of truss topology design . . . . .	252
4.4.1	Combined truss topology and geometry optimization . . . . .	252
4.4.2	Truss design with buckling constraints . . . . .	255

4.4.3	Control of free vibrations .....	256
4.4.4	Variations of the theme .....	258
<b>5</b>	<b>Appendices .....</b>	<b>261</b>
5.1	Appendix: Matlab codes .....	261
5.1.1	A 99 line topology optimization code for compliance minimization .....	261
5.1.2	Matlab implementation .....	262
5.1.3	Extensions .....	264
5.1.4	Matlab code .....	267
5.1.5	A 105 line MATLAB code for compliant mechanism synthesis .....	269
5.1.6	A 91 line MATLAB code for heat conduction problems	270
5.2	Appendix: The existence issue .....	272
5.2.1	Variable thickness sheet design: Existence .....	272
5.2.2	Density design with a gradient constraint: Existence ..	274
5.3	Appendix: Aspects of shape design: The boundary variations method .....	276
5.3.1	Design parametrization in shape design .....	276
5.3.2	The basics of a boundary shape design method .....	277
5.4	Appendix: Homogenization and layered materials .....	280
5.4.1	The homogenization formulas .....	281
5.4.2	The smear-out process .....	283
5.4.3	The moment formulation .....	287
5.4.4	Stress criteria for layered composites .....	291
5.4.5	Homogenization formulas for Kirchhoff plates .....	295
5.4.6	Hashin-Shtrikman-Walpole (HSW) bounds .....	296
5.5	Appendix: Barrier methods for topology design .....	298
5.5.1	Notation .....	298
5.5.2	Interior-point methods .....	299
5.5.3	A barrier method for topology optimization .....	301
5.5.4	The free material multiple load case as a SDP problem	302
<b>6</b>	<b>Bibliographical notes .....</b>	<b>305</b>
6.1	Books and survey papers .....	305
6.2	Papers .....	307
	<b>References .....</b>	<b>319</b>
	<b>Author index .....</b>	<b>355</b>
	<b>Index .....</b>	<b>365</b>