

Contents

Preface	V
List of contributors	XIII
1. Basic notions on substitutions (by V. Berthé and A. Siegel)	1
1.1 Word combinatorics	2
1.2 Substitutions	7
1.3 Automata	12
1.4 Topological and measure-theoretic dynamical systems	16
1.5 Spectral theory	21
1.6 Factors of substitutive dynamical systems	26

Part I. Arithmetics and combinatorics of substitutions

2. Substitutions, arithmetic and finite automata: an introduction (by C. Mauduit)	35
2.1 The Morse sequence	35
2.2 The Rudin-Shapiro sequence	41
2.3 The Baum-Sweet sequence	47
2.4 The Cantor sequence	49
2.5 An application of substitutions to criteria of divisibility	49
2.6 The Fibonacci sequence	51
3. Automatic sequences and transcendence (by V. Berthé) ..	53
3.1 Introduction	54
3.2 The Christol, Kamae, Mendès France, and Rauzy theorem ..	56
3.3 Transcendence in the real case and continued fractions ..	64
3.4 Some functions defined by Carlitz	66
3.5 Some examples of automaton proofs	72
3.6 Conclusion	79
4. Substitutions and partitions of the set of positive integers (by J. -I. Tamura)	81
4.1 Beatty and Sturmian sequences, and associated partitions ..	82
4.2 Partitions given by substitutions	85

4.3	Similis partitions	87
4.4	Log-fixed points and Kolakoski words	93
4.5	Problems	97
<hr/>		
Part II. Dynamics of substitutions		
<hr/>		
5.	Substitutions and symbolic dynamical systems (by S. Ferenczi)	101
5.1	The Morse sequence: elementary properties	101
5.2	The Morse sequence: advanced properties	114
5.3	The Rudin-Shapiro sequence	125
5.4	The Fibonacci sequence	128
5.5	The Chacon sequence	133
6.	Sturmian Sequences (by P. Arnoux)	143
6.1	Sturmian sequences. Basic properties	145
6.2	Biinfinite Sturmian sequences	156
6.3	Coding sequences for Sturmian sequences and Sturmian systems	160
6.4	Sturmian sequences: arithmetic properties; continued fractions	175
6.5	Sturmian substitutions. Dynamical interpretations	183
6.6	Natural extension for the recoding of Sturmian sequences	189
6.7	Miscellaneous remarks	195
7.	Spectral theory and geometric representation of substitutions (by A. Siegel)	199
7.1	Shifts of finite type: introduction	202
7.2	Substitutive dynamical systems and shifts of finite type	209
7.3	Spectral theory of substitutive dynamical systems	219
7.4	The Rauzy fractal	231
7.5	Geometric realization of substitutions of Pisot type	237
7.6	Extensions and applications	250
8.	Diophantine approximations, substitutions, and fractals (by S. Ito)	253
8.1	Substitutions and domains with fractal boundary	254
8.2	Generalized substitutions	257
8.3	Dynamical systems associated with the stepped surface	270
8.4	Renormalization and realization of substitutive systems	278
8.5	Fractal boundary	282
8.6	Continued fraction expansions and substitutions	285
8.7	Diophantine applications	289

Part III. Extensions to free groups and interval transformations

9. Infinite words generated by invertible substitutions (by Z.-Y. Wen)	295
9.1 Preliminary	296
9.2 Structure of invertible substitutions	300
9.3 Singular words of the Fibonacci word and applications	303
9.4 Properties of factors of the fixed points of invertible substitutions	312
10. Polynomial dynamical systems associated with substitutions (by J. Peyrière)	321
10.1 Polynomial identities in the algebra of 2×2 -matrices	323
10.2 Trace maps	330
10.3 The case of 3×3 -matrices	338
10.4 Comments	341
11. Piecewise linear transformations of the unit interval and Cantor sets (by M. Mori)	343
11.1 Definitions	344
11.2 Ergodic properties of piecewise Markov transformations	346
11.3 Non-Markov transformation: β -expansion	353
11.4 Cantor sets	354
12. Some open problems (by P. Arnoux and V. Berthé)	363
12.1 The S -adic conjecture	363
12.2 Multidimensional continued fraction expansions	366
12.3 Combinatorics on two-dimensional words	370
12.4 Substitutions, rotations and toral automorphisms	372
12.5 Arithmetics in $SL(d, \mathbb{N})$ and $SL(d, \mathbb{Z})$	373
A. Undecomposable matrices in dimension 3 (by J. Rivat)	375
References	377
Index	399