

Contents

1	Introduction	1
A	From Statistics to Geostatistics	7
2	Mean, Variance, Covariance	9
	The mean: center of mass	9
	Distribution function	11
	Expectation	12
	Variance	13
	Covariance	14
3	Linear Regression and Simple Kriging	15
	Experimental covariance	15
	Linear regression	16
	Variance-covariance matrix	20
	Multiple linear regression	21
	Simple kriging	24
4	Kriging the Mean	27
	Arithmetic mean and its estimation variance	27
	Estimating the mean with spatial correlation	28
	No systematic bias	29
	Variance of the estimation error	30
	Minimal estimation variance	30
	Kriging equations	31
	Case of no correlation	32
B	Geostatistics	35
5	Regionalized Variable and Random Function	39
	Multivariate time/space data	39
	Regionalized variable	40
	Random variable and regionalized value	41

Random function	41
Probability distributions	42
Strict stationarity	43
Stationarity of first two moments	44
6 Variogram Cloud	45
Dissimilarity versus separation	45
Experimental variogram	47
Replacing the experimental by a theoretical variogram	48
7 Variogram and Covariance Function	50
Regional variogram	50
Theoretical variogram	50
Covariance function	52
Positive definite function	53
Conditionally negative definite function	53
Fitting the variogram with a covariance function	55
8 Examples of Covariance Functions	57
Nugget-effect model	57
Exponential covariance function	57
Spherical model	58
Derivation of the spherical covariance	58
9 Anisotropy	62
Geometric Anisotropy	62
Rotating and dilating an ellipsoid	62
Exploring 3D space for anisotropy	64
Zonal anisotropy	65
Nonlinear deformations of space	65
10 Extension and Dispersion Variance	66
Support	66
Extension variance	67
Dispersion variance	68
Krige's relation	69
Change of support effect	70
Change of support: affine model	71
Application: acoustic data	73
Comparison of sampling designs	76
11 Ordinary Kriging	79
Ordinary kriging problem	79
Simple kriging of increments	81
Block kriging	82

Simple kriging with an estimated mean	84
Kriging the residual	85
Cross validation	87
Kriging with known measurement error variance	88
12 Kriging Weights	89
Geometry	89
Geometric anisotropy	91
Relative position of samples	91
Screen effect	92
Factorizable covariance functions	93
Negative kriging weights	94
13 Mapping with Kriging	96
Kriging for spatial interpolation	96
Neighborhood	97
14 Linear Model of Regionalization	101
Spatial anomalies	101
Nested variogram model	102
Decomposition of the random function	103
Second-order stationary regionalization	104
Intrinsic regionalization	105
Intrinsic regionalization with mostly stationary components	105
Locally stationary regionalization	106
15 Kriging Spatial Components	107
Kriging of the intrinsic component	107
Kriging of a second-order stationary component	108
Filtering	110
Application: kriging spatial components of arsenic data	111
16 The Smoothness of Kriging	113
Kriging with irregularly spaced data	113
Sensitivity to choice of variogram model	115
Application: kriging topographic data	117
C Multivariate Analysis	121
17 Principal Component Analysis	123
Uses of PCA	123
Transformation into factors	123
Maximization of the variance of a factor	125
Interpretation of the factor variances	126
Correlation of the variables with the factors	127

18 Canonical Analysis	137
Factors in two groups of variables	137
Intermezzo: singular value decomposition	138
Maximization of the correlation	138
19 Correspondence Analysis	140
Disjunctive table	140
Contingency table	140
Canonical analysis of disjunctive tables	141
Coding of a quantitative variable	141
Contingencies between two quantitative variables	141
Continuous correspondence analysis	142
D Multivariate Geostatistics	143
20 Direct and Cross Covariances	145
Cross covariance function	145
Delay effect	145
Cross variogram	146
Pseudo cross variogram	149
Difficult characterization of the cross covariance function	150
21 Covariance Function Matrices	151
Covariance function matrix	151
Cramer's theorem	151
Spectral densities	152
Phase shift	153
22 Intrinsic Multivariate Correlation	154
Intrinsic correlation model	154
Linear model	155
Codispersion coefficients	156
23 Heterotopic Cokriging	158
Isotopy and heterotopy	158
Ordinary cokriging	159
Simple cokriging	161
24 Collocated Cokriging	165
Cokriging neighborhood	165
Collocated simple cokriging	166
Collocated ordinary cokriging	167
Simplification with a particular covariance model	168

25 Isotopic Cokriging	170
Cokriging with isotopic data	170
Autokrigability	171
Bivariate ordinary cokriging	173
26 Multivariate Nested Variogram	175
Linear model of coregionalization	175
Bivariate fit of the experimental variograms	177
Multivariate fit	178
The need for an analysis of the coregionalization	181
27 Case Study: Ebro Estuary	183
Kriging conductivity	183
Cokriging of chlorophyll	185
Conditional simulation of chlorophyll	187
28 Coregionalization Analysis	194
Regionalized principal component analysis	194
Generalizing the analysis	195
Regionalized canonical and redundancy analysis	196
Cokriging regionalized factors	196
Regionalized multivariate analysis	197
29 Kriging a Complex Variable	200
Coding directional data as a complex variable	200
Complex covariance function	200
Complex kriging	201
Cokriging of the real and imaginary parts	202
Complex kriging and cokriging versus a separate kriging	203
Complex covariance function modeling	205
30 Bilinear Coregionalization Model	207
Complex linear model of coregionalization	207
Bilinear model of coregionalization	208
E Selective Geostatistics	211
31 Thresholds and Selectivity Curves	213
Threshold and proportion	213
Tonnage, recovered quantity, investment and profit	213
Selectivity	216
Recovered quantity as a function of tonnage	217
Time series in environmental monitoring	219

32 Lognormal Estimation	221
Information effect and quality of estimators	221
Logarithmic Gaussian model	223
Moments of the lognormal variable	224
Lognormal simple kriging	226
Proportional effect	228
Permanence of lognormality	228
Stable variogram model	233
Lognormal point and block ordinary kriging	233
33 Gaussian Anamorphosis with Hermite Polynomials	238
Gaussian anamorphosis	238
Hermite polynomials	239
Expanding a function into Hermite polynomials	240
Probabilistic interpretation	240
Moments of a function of a Gaussian variable	241
Conditional expectation of a function of a Gaussian variable	243
Empirical Gaussian anamorphosis	244
Smoothing the empirical anamorphosis	245
Bijectivity of Gaussian anamorphosis	247
34 Isofactorial Models	250
Isofactorial bivariate distribution	250
Isofactorial decomposition	251
Isofactorial models	252
Choice of marginal and of isofactorial bivariate distribution	253
Hermitian and Laguerre isofactorial distributions	254
Intermediate types between diffusion and mosaic models	258
35 Isofactorial Change of Support	262
The point-block-panel problem	262
Cartier's relation and point-block correlation	262
Discrete Gaussian point-block model	266
General structure of isofactorial change-of-support	267
36 Kriging with Discrete Point-Bloc Models	273
Non-linear function of a block variable	273
Conditional expectation and disjunctive kriging of a bloc	274
Disjunctive kriging of a panel	275
Uniform conditioning	277

F Non-Stationary Geostatistics	281
37 External Drift	283
Depth measured with drillholes and seismic	283
Estimating with a shape function	284
Estimating external drift coefficients	285
Cross validation with external drift	290
Regularity of the external drift function	294
Cokriging with multiple external drift	296
Ebro estuary: numerical model output as external drift	297
Comparing results of conditional simulations and kriging	297
38 Universal Kriging	300
Universal kriging system	300
Estimation of the drift	302
Underlying variogram and estimated residuals	303
From universal to intrinsic kriging	306
39 Translation Invariant Drift	308
Exponential-polynomial basis functions	308
Intrinsic random functions of order k	309
Generalized covariance function	310
Intrinsic kriging	311
Trigonometric temporal drift	312
Filtering trigonometric temporal drift	312
Dual kriging	313
Splines	314
APPENDIX	317
Matrix Algebra	319
Linear Regression Theory	329
Covariance and Variogram Models	334
Additional Exercises	337
Solutions to Exercises	339
References	353
Bibliography	358
Index	381