

---

## Contents

<b>1</b>	<b>Physical Background</b> .....	1
1.1	The Double-Slit Experiment .....	1
1.2	Wave Functions .....	3
1.3	State Space .....	4
1.4	The Schrödinger Equation .....	4
1.5	Mathematical Supplement: Operators on Hilbert Spaces .....	6
<b>2</b>	<b>Dynamics</b> .....	11
2.1	Conservation of Probability .....	11
2.2	Existence of Dynamics .....	12
2.3	The Free Propagator .....	16
2.4	Mathematical Supplement: Operator Adjoints .....	17
2.5	Mathematical Supplement: the Fourier Transform .....	20
2.5.1	Definition of the Fourier Transform .....	20
2.5.2	Properties of the Fourier Transform .....	21
2.5.3	Functions of the Derivative .....	22
<b>3</b>	<b>Observables</b> .....	25
3.1	Mean Values and the Momentum Operator .....	25
3.2	Observables .....	26
3.3	The Heisenberg Representation .....	27
3.4	Quantization .....	28
3.5	Pseudodifferential Operators .....	29
<b>4</b>	<b>The Uncertainty Principle</b> .....	31
4.1	The Heisenberg Uncertainty Principle .....	31
4.2	A Refined Uncertainty Principle .....	32
4.3	Application: Stability of Hydrogen .....	33

<b>5</b>	<b>Spectral Theory</b> .....	35
5.1	The Spectrum of an Operator .....	35
5.2	Functions of Operators and the Spectral Mapping Theorem ..	40
5.3	Applications to Schrödinger Operators .....	42
5.4	Spectrum and Evolution .....	47
5.5	Variational Characterization of Eigenvalues .....	49
5.6	Number of Bound States .....	54
5.7	Mathematical Supplement: Integral Operators .....	60
<b>6</b>	<b>Scattering States</b> .....	61
6.1	Short-range Interactions: $\mu > 1$ .....	62
6.2	Long-range Interactions: $\mu \leq 1$ .....	65
6.3	Wave Operators .....	65
<b>7</b>	<b>Special Cases</b> .....	69
7.1	The Infinite Well .....	69
7.2	The Torus .....	70
7.3	A Potential Step .....	70
7.4	The Square Well .....	72
7.5	The Harmonic Oscillator .....	73
7.6	A Particle on a Sphere .....	76
7.7	The Hydrogen Atom .....	76
7.8	A Particle in an External EM Field .....	79
<b>8</b>	<b>Many-particle Systems</b> .....	81
8.1	Quantization of a Many-particle System .....	81
8.2	Separation of the Centre-of-mass Motion .....	85
8.3	Break-ups .....	87
8.4	The HVZ Theorem .....	88
8.5	Intra- vs. Inter-cluster Motion .....	90
8.6	Existence of Bound States for Atoms and Molecules .....	92
8.7	Scattering States .....	93
8.8	Mathematical Supplement: Tensor Products .....	95
8.9	Appendix: Hartree and Gross-Pitaevski Equations .....	97
<b>9</b>	<b>Density Matrices</b> .....	103
9.1	Introduction .....	103
9.2	States and Dynamics .....	103
9.3	Open Systems .....	105
9.4	The Thermodynamic Limit .....	106
9.5	Equilibrium States .....	107
9.6	The $T \rightarrow 0$ Limit .....	107
9.7	Example: a System of Harmonic Oscillators .....	109
9.8	A Particle Coupled to a Reservoir .....	110
9.9	Quantum Systems .....	111

9.10	Problems	111
9.11	Hilbert Space Approach	111
9.12	BEC at T=0	113
9.13	Appendix: the Ideal Bose Gas	114
9.14	Appendix: Bose-Einstein Condensation	119
9.15	Mathematical Supplement: the Trace, and Trace Class Operators	122
9.16	Mathematical Supplement: Projections	127
<b>10</b>	<b>Perturbation Theory: Feshbach Method</b>	<b>131</b>
10.1	The Feshbach Method	132
10.2	Example: The Zeeman Effect	135
10.3	Example: Time-dependent Perturbations	137
10.4	Appendix: Proof of Theorem 10.1	142
<b>11</b>	<b>The Feynman Path Integral</b>	<b>145</b>
11.1	The Feynman Path Integral	145
11.2	Generalizations of the Path Integral	148
11.3	Mathematical Supplement: The Trotter Product Formula	149
<b>12</b>	<b>Quasi-classical Analysis</b>	<b>151</b>
12.1	Quasi-classical Asymptotics of the Propagator	152
12.2	Quasi-classical Asymptotics of Green's Function	156
12.2.1	Appendix	159
12.3	Bohr-Sommerfeld Semi-classical Quantization	159
12.4	Quasi-classical Asymptotics for the Ground State Energy	161
12.5	Mathematical Supplement: Operator Determinants	163
<b>13</b>	<b>Mathematical Supplement: The Calculus of Variations</b>	<b>167</b>
13.1	Functionals	167
13.2	The First Variation and Critical Points	169
13.3	Constrained Variational Problems	173
13.4	The Second Variation	174
13.5	Conjugate Points and Jacobi Fields	176
13.6	The Action of the Critical Path	179
13.7	Appendix: Connection to Geodesics	182
<b>14</b>	<b>Resonances</b>	<b>185</b>
14.1	Tunneling and Resonances	185
14.2	The Free Resonance Energy	187
14.3	Instantons	189
14.4	Positive Temperatures	191
14.5	Pre-exponential Factor for the Bounce	193
14.6	Contribution of the Zero-mode	195
14.7	Bohr-Sommerfeld Quantization for Resonances	195

<b>15</b>	<b>Introduction to Quantum Field Theory</b> .....	199
15.1	The Place of QFT .....	199
15.1.1	Physical Theories .....	200
15.1.2	The Principle of Minimal Action .....	200
15.2	Klein-Gordon Theory as a Hamiltonian System .....	201
15.2.1	The Legendre Transform .....	201
15.2.2	Hamiltonians .....	202
15.2.3	Poisson Brackets .....	202
15.2.4	Hamilton's Equations .....	204
15.3	Maxwell's Equations as a Hamiltonian System .....	205
15.4	Quantization of the Klein-Gordon and Maxwell Equations ...	207
15.4.1	The Quantization Procedure .....	208
15.4.2	Creation and Annihilation Operators .....	212
15.4.3	Wick Ordering .....	214
15.4.4	Quantizing Maxwell's Equations .....	215
15.5	Fock Space .....	216
15.6	Generalized Free Theory .....	219
15.7	Interactions .....	220
15.8	Quadratic Approximation .....	222
15.8.1	Further Discussion .....	228
15.8.2	A Brief Remark on Many-body Hamiltonians in Second Quantization and the Hartree Approximation ..	229
<b>16</b>	<b>Quantum Electrodynamics of Non-relativistic Particles: The Theory of Radiation</b> .....	231
16.1	The Hamiltonian .....	231
16.2	Perturbation Set-up .....	234
16.3	Results .....	236
16.4	Mathematical Supplements .....	239
16.4.1	Spectral Projections .....	239
16.4.2	Projecting-out Procedure .....	240
<b>17</b>	<b>Supplement: Renormalization Group</b> .....	241
17.1	The Decimation Map .....	242
17.2	Relative Bounds .....	243
17.3	Elimination of Particle and High Photon Energy Degrees of Freedom .....	244
17.4	Generalized Normal Form of Operators on Fock Space .....	248
17.5	The Hamiltonian $H_0(\varepsilon, z)$ .....	250
17.6	A Banach Space of Operators .....	254
17.7	Rescaling .....	255
17.8	The Renormalization Map .....	257
17.9	Linearized Flow .....	258
17.10	Central-stable Manifold for RG and Spectra of Hamiltonians	261
17.11	Appendix .....	265

<b>18 Comments on Missing Topics, Literature, and Further Reading .....</b>	<b>267</b>
<b>References .....</b>	<b>273</b>
<b>Index .....</b>	<b>281</b>