

Contents

1	Introduction	1
1.1	The nature of the problems	2
1.2	The combinatorial structures in question	4
1.2.1	Designs	4
1.2.2	Difference Sets	6
1.2.3	Projective planes and planar functions	7
1.2.4	Projective geometries and Singer difference sets	9
1.2.5	Hadamard matrices and weighing matrices	10
1.2.6	Irreducible cyclic codes, two-intersection sets and sub-difference sets	11
1.3	Group rings, characters, Fourier analysis	14
1.4	Number theoretic tools	19
1.5	Algebraic-combinatorial tools	24
2	The field descent	27
2.1	The fixing theorem	27
2.2	Prescribed absolute value	31
2.3	Bounding the absolute value	36
2.4	The modulus equation and class groups	37
2.4.1	Class groups of cyclotomic fields	39
2.4.2	Class groups of CM -fields	44
2.4.3	p -ranks and class fields towers	48
3	Exponent bounds	53
3.1	Self-conjugacy exponent bounds	53
3.1.1	Turyn's exponent bound	54
3.1.2	The coset intersection lemma	55
3.1.3	McFarland difference sets	57
3.1.4	Semiregular relative difference sets	58
3.1.5	Two recent families of difference sets	61
3.1.6	Chen difference sets	62
3.1.7	Davis-Jedwab difference sets	66
3.2	Field descent exponent bounds	67

3.2.1	A general exponent bound for difference sets	68
3.2.2	Difference sets with $\gcd(v, n) > 1$	69
3.2.3	Towards Ryser's conjecture	71
3.2.4	Circulant Hadamard matrices and Barker sequences	73
3.2.5	Relative difference sets and planar functions	74
3.2.6	Group invariant weighing matrices	77
4	Two-weight irreducible cyclic codes	79
4.1	A necessary and sufficient condition	80
4.2	All two-weight irreducible cyclic codes?	83
4.2.1	Subfield and semiprimitive codes	83
4.2.2	The exceptional codes	84
4.3	Partial proof of Conjecture 4.2.4	85
4.4	Two-intersection sets and sub-difference sets	88
4.4.1	Two-intersection sets in $\text{PG}(m-1, q)$	88
4.4.2	Sub-difference sets of Singer difference sets	88
	Bibliography	91
	Index	99