Table of Contents

1 Model Problem and Its Discretization 1
- 1.1 Conservation Laws 1
- 1.2 Finite Volume Methods 6

2 Multiscale Setting 11
- 2.1 Hierarchy of Meshes 11
- 2.2 Motivation 13
- 2.3 Box Wavelet 17
  - 2.3.1 Box Wavelet on a Cartesian Grid Hierarchy 17
  - 2.3.2 Box Wavelet on an Arbitrary Nested Grid Hierarchy 19
- 2.4 Change of Stable Completion 22
- 2.5 Box Wavelet with Higher Vanishing Moments 24
  - 2.5.1 Definition and Construction 24
  - 2.5.2 A Univariate Example 26
  - 2.5.3 A Remark on Compression Rates 29
- 2.6 Multiscale Transformation 29

3 Locally Refined Spaces 33
- 3.1 Adaptive Grid and Significant Details 34
- 3.2 Grading 36
- 3.3 Local Multiscale Transformation 44
- 3.4 Grading Parameter 47
- 3.5 Locally Uniform Grids 52
- 3.6 Algorithms: Encoding, Thresholding, Grading, Decoding 55
- 3.7 Conservation Property 60
- 3.8 Application to Curvilinear Grids 62

4 Adaptive Finite Volume Scheme 73
- 4.1 Construction 73
  - 4.1.1 Strategies for Local Flux Evaluation 75
  - 4.1.2 Strategies for Prediction of Details 77
- 4.2 Algorithms: Initial data, Prediction, Fluxes and Evolution 82

5 Error Analysis ... 89
5.1 Perturbation Error ... 90
5.2 Stability of Approximation ... 93
5.3 Reliability of Prediction ... 97

6 Data Structures and Memory Management ... 113
6.1 Algorithmic Requirements and Design Criteria ... 113
6.2 Hashing ... 115
6.3 Data Structures ... 118

7 Numerical Experiments ... 123
7.1 Parameter Studies ... 123
7.1.1 Test Configurations ... 124
7.1.2 Discretization ... 126
7.1.3 Computational Complexity and Stability ... 127
7.1.4 Hash Parameters ... 131
7.2 Real World Application ... 133
7.2.1 Configurations ... 133
7.2.2 Discretization ... 134
7.2.3 Discussion of Results ... 136

A Plots of Numerical Experiments ... 139

B The Context of Biorthogonal Wavelets ... 151
B.1 General Setting ... 151
B.1.1 Multiscale Basis ... 152
B.1.2 Stable Completion ... 153
B.1.3 Multiscale Transformation ... 154
B.2 Biorthogonal Wavelets of the Box Function ... 157
B.2.1 Haar Wavelets ... 157
B.2.2 Biorthogonal Wavelets on the Real Line ... 158

References ... 163

List of Figures ... 169

List of Tables ... 171

Notation ... 173

Index ... 179