

# Table of Contents

<b>Preface</b> .....	V
<b>On Kolmogorov's equations for finite dimensional diffusions</b>	
N.V. Krylov .....	1
1. Solvability of It's stochastic equations .....	1
2. Markov property of solutions .....	8
2.1 Regular equations .....	8
2.2 Some properties of Euler's approximations .....	11
2.3 Markov property .....	15
3. Conditional version of Kolmogorov's equation .....	16
4. Differentiability of solutions of stochastic equations with respect to initial data .....	21
4.1 Estimating moments of solutions of It's equations .....	22
4.2 Smoothness of solutions depending on a parameter .....	26
4.3 Estimating moments of derivatives of solutions .....	31
4.4 The notions of $L$ -continuity and $L$ -differentiability .....	33
4.5 Differentiability of certain expectations depending on a parameter	36
5. Kolmogorov's equation in the whole space .....	42
5.1 Stratified equations .....	43
5.2 Sufficient conditions for regularity .....	46
5.3 Kolmogorov's equation .....	48
6. Some integral approximations of differential operators .....	53
7. Kolmogorov's equations in domains .....	58
<b><math>L^p</math>-analysis of finite and infinite dimensional diffusion operators</b>	
Michael Röckner .....	65
1. Introduction .....	65
2. Solution of Kolmogorov equations via sectorial forms .....	66
2.1 Preliminaries .....	66
2.2 Sectorial forms .....	68
2.3 Sectorial forms on $L^2(E; m)$ .....	70
2.4 Examples and Applications .....	72

3.	Symmetrizing measures .....	78
3.1	The classical finite dimensional case .....	78
3.2	Representation of symmetric diffusion operators .....	80
3.3	Ornstein–Uhlenbeck type operators. ....	81
3.4	Operators with non–linear drift. ....	83
4.	Non-sectorial cases: perturbations by divergence free vector fields ...	86
4.1	Diffusion operators on $L^p(E; m)$ .....	86
4.2	Solution of Kolmogorov equations on $L^1(E; m)$ .....	88
4.3	Uniqueness problem. ....	92
4.4	Concluding remarks. ....	95
5.	Invariant measures: regularity, existence and uniqueness .....	96
5.1	Sectorial case .....	96
5.2	Non–sectorial cases .....	99
6.	Corresponding diffusions and relation to Martingale problems .....	103
6.1	Existence of associated diffusions .....	103
6.2	Solution of the martingale problem .....	105
6.3	Uniqueness .....	105
7.	Appendix .....	106
7.1	Kolmogorov equations in $L^2(E; \mu)$ for infinite dimensional manifolds $E$ : a case study from continuum statistical mechanics ....	106
7.2	Ergodicity .....	110

## Parabolic equations on Hilbert spaces

J. Zabczyk .....	117	
1.	Preface .....	117
2.	Preliminaries .....	119
2.1	Linear operators .....	119
2.2	Measures and random variables .....	123
2.3	Wiener process and stochastic equations .....	127
3.	Heat Equation .....	130
3.1	Introduction .....	131
3.2	Regular initial functions .....	135
3.3	Gross Laplacian .....	137
3.4	Heat equation with general initial functions .....	139
3.5	Generators of the heat semigroups .....	143
3.6	Nonparabolicity .....	147
4.	Transition semigroups .....	149
4.1	Transition semigroups in the space of continuous functions ....	150
4.2	Transition semigroups in spaces of square summable functions ..	154
5.	Heat equation with a first order term .....	157
5.1	Introduction .....	158
5.2	Regular initial functions .....	159
5.3	General initial functions .....	163
5.4	Range condition and examples .....	170

6. General parabolic equations. Regularity .....	174
6.1 Convolution type and evaluation maps .....	174
6.2 Solutions of stochastic equations .....	178
6.3 Space and time regularity of generalized solutions .....	179
6.4 Strong Feller property .....	181
7. General parabolic equations. Uniqueness. ....	186
7.1 Uniqueness for the heat equation .....	186
7.2 Uniqueness in the general case .....	187
8. Parabolic equations in open sets .....	191
8.1 Introduction .....	191
8.2 Main theorem .....	192
8.3 Estimates of the exit probabilities .....	195
9. Applications .....	198
9.1 HJB equation of stochastic control .....	198
9.2 Solvability of HJB equation .....	202
9.3 Kolmogorov's equation in mathematical finance .....	204
10. Appendix .....	206
10.1 Implicit function theorems .....	206